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Section 4.9

Antiderivatives

Antiderivatives

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example 1

Find an antiderivative of $f(x) = 3x^2$.

General Antiderivatives

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Example 2

Find the general antiderivative of each given function.

$$f(x) = x$$

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = x^n$$

Example 3

Find the general antiderivative of each given function.

$$f(x) = x^{3/2}$$

$$g(x) = x^{-3}$$

$$h(x) = x^{-1}$$

The Power Rule

If $f(x) = x^n$ where $n \neq -1$, then the general antiderivative of $f(x)$ is

$$F(x) = \frac{x^{n+1}}{n+1} + C.$$

Example 4 – Trigonometric Antiderivatives

Find the antiderivative of each trigonometric function.

$$f(x) = \sin x$$

$$g(x) = \cos x$$

$$h(x) = \sec x \tan x$$

$$f(x) = \csc x \cot x$$

$$g(x) = \sec^2 x$$

$$h(x) = \csc^2 x$$

Example 5

Find $f(x)$ given $f'(x) = 8x^3 - 4x^2 + 7$, $f(0) = 12$.

Indefinite Integrals

If $F(x)$ is any antiderivative of $f(x)$, then the indefinite integral of $f(x)$ with respect to x is

$$\int f(x)dx = F(x) + C$$

where C is an arbitrary constant.

In other words, calculating the indefinite integral of a function is the same as calculating the general antiderivative of the function.

Example 6

Compute each indefinite integral.

$$\int \frac{12x^8 - x^{1/2}}{x^3} dx$$

$$\int y(y-1)^2 dy$$

$$\int (1 + \cot^2\theta) d\theta$$

Example 7

Find $f(x)$ given $f''(x) = 2x^3 + 3x^2 - 4x + 5$, $f(0) = 2$, and $f(1) = 0$.