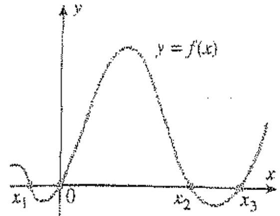


Figure 1.1.6 f has zeros at x_1, 0, x_2, and x_3.

which is impossible, since f cannot assign two different values to a. Thus, there is no function f whose graph is the given curve. This illustrates the following general result, which we will call the ***vertical line test***.

> **1.1.3 THE VERTICAL LINE TEST.** *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*

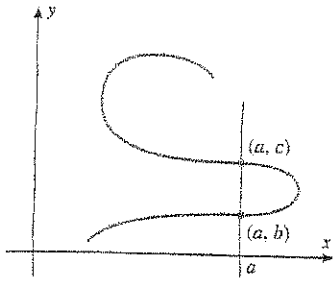


Figure 1.1.7 This curve cannot be the graph of a function.

▶ Example 3 The graph of the equation

$$x^2 + y^2 = 25 \tag{1}$$

is a circle of radius 5 centered at the origin and hence there are vertical lines that cut the graph more than once. This can also be seen algebraically by solving Equation (1) for y in terms of x:

$$y = \pm\sqrt{25 - x^2}$$

This equation does not define y as a function of x because the right side is "multiple valued" in the sense that values of x in the interval $(-5, 5)$ produce two corresponding values of y. For example, if $x = 4$, then $y = \pm 3$, and hence $(4, 3)$ and $(4, -3)$ are two points on the circle that lie on the same vertical line (Figure 1.1.8). However, we can regard the circle as the union of the two semicircles

$$y = \sqrt{25 - x^2} \quad \text{and} \quad y = -\sqrt{25 - x^2}$$

each of which defines y as a function of x (Figure 1.1.9). ◀

See Web Appendix G for a review of circles.

O'Neal
H: (816) 763-6867
I have caller ID
E-mail
joneal@raypec.k12.mo.us
or
joe.oneal@mcckc.edu

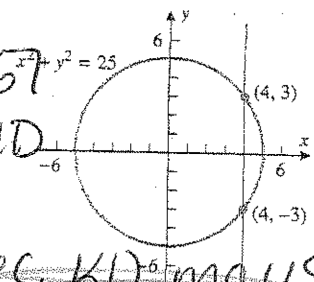


Figure 1.1.8

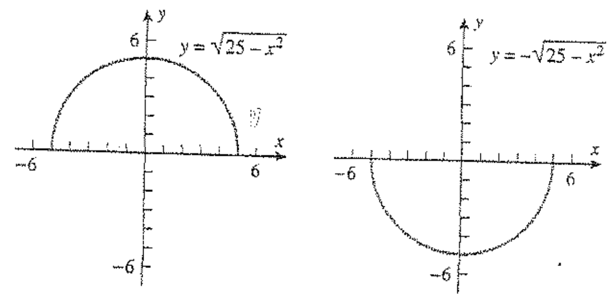


Figure 1.1.9 The union of these semicircles is the full circle.

THE ABSOLUTE VALUE FUNCTION

Recall that the ***absolute value*** or ***magnitude*** of a real number x is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Symbols such as $+x$ and $-x$ are deceptive, since it is tempting to conclude that $+x$ is positive and $-x$ is negative. However, this need not be so, since x itself can be positive or negative. For example, if x is negative, say $x = -3$, then $-x = 3$ is positive and $+x = -3$ is negative.

The effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative. Thus,

$$|5| = 5, \quad \left|-\tfrac{4}{7}\right| = \tfrac{4}{7}, \quad |0| = 0$$

A more detailed discussion of the properties of absolute value is given in Web Appendix E. However, for convenience we provide the following summary of its algebraic properties.

There will be 3 help sessions this summer at LV.
Details later.

For each input x, the corresponding output y is obtained by substituting x in this formul
For example,

$$f(0) = 3(0)^2 - 4(0) + 2 = 2 \qquad \boxed{f \text{ associates } y = 2 \text{ with } x = 0.}$$

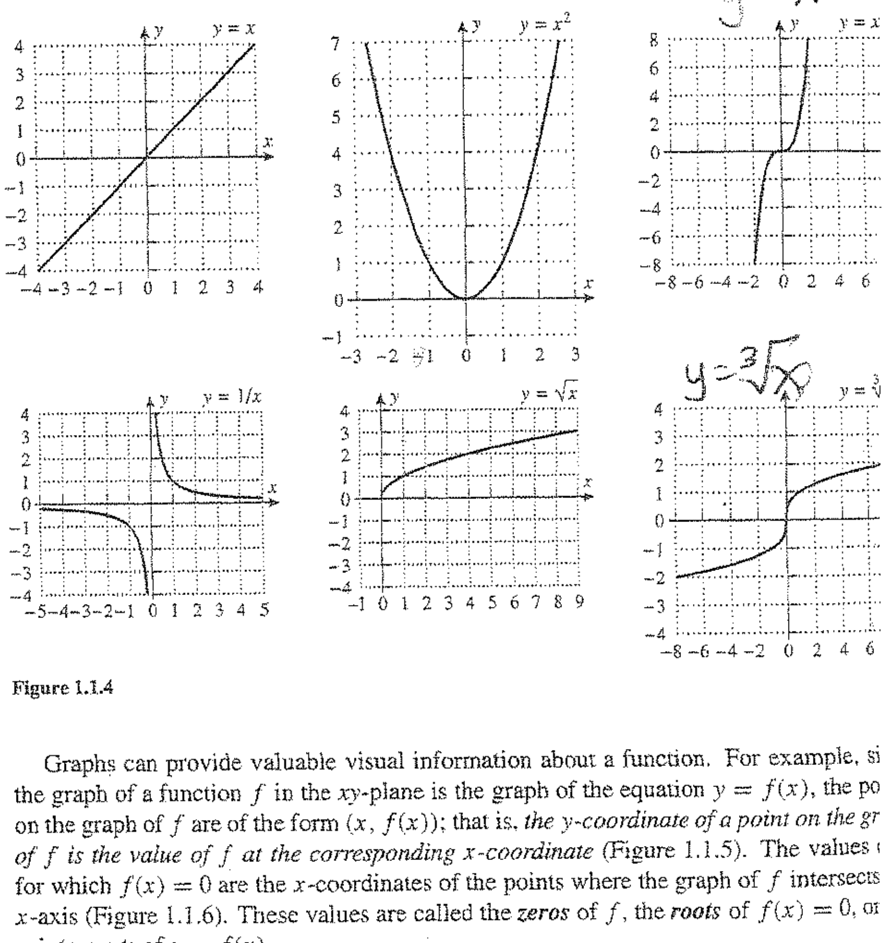
$$f(-1.7) = 3(-1.7)^2 - 4(-1.7) + 2 = 17.47 \qquad \boxed{f \text{ associates } y = 17.47 \text{ with } x = -1.7.}$$

$$f(\sqrt{2}) = 3(\sqrt{2})^2 - 4\sqrt{2} + 2 = 8 - 4\sqrt{2} \qquad \boxed{f \text{ associates } y = 8 - 4\sqrt{2} \text{ with } x = \sqrt{2}.}$$

GRAPHS OF FUNCTIONS

If f is a real-valued function of a real variable, then the ***graph*** of f in the xy-plane is defin to be the graph of the equation $y = f(x)$. For example, the graph of the function $f(x) =$ is the graph of the equation $y = x$, shown in Figure 1.1.4. That figure also shows the grap of some other basic functions that may already be familiar to you. In the next section will discuss techniques for graphing functions using graphing technology.

$y = x^3$



Since $\sqrt{x}$ is imaginary for negative values of x, there are no points on the graph of $y = \sqrt{x}$ in the region where $x < 0$.

Figure 1.1.4

Graphs can provide valuable visual information about a function. For example, si the graph of a function f in the xy-plane is the graph of the equation $y = f(x)$, the po on the graph of f are of the form $(x, f(x))$; that is, *the y-coordinate of a point on the gr of f is the value of f at the corresponding x-coordinate* (Figure 1.1.5). The values for which $f(x) = 0$ are the x-coordinates of the points where the graph of f intersects x-axis (Figure 1.1.6). These values are called the ***zeros*** of f, the ***roots*** of $f(x) = 0$, or ***x-intercepts*** of $y = f(x)$.

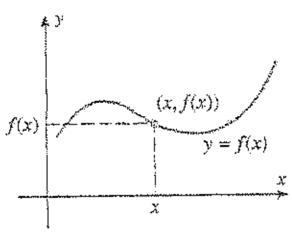


Figure 1.1.5 The y-coordinate of a point on the graph of $y = f(x)$ is the value of f at the corresponding x-coordinate.

THE VERTICAL LINE TEST

Not every curve in the xy-plane is the graph of a function. For example, consider the c in Figure 1.1.7, which is cut at two distinct points, (a, b) and (a, c), by a vertical line. curve cannot be the graph of $y = f(x)$ for any function f; otherwise, we would have

$$f(a) = b \quad \text{and} \quad f(a) = c$$

1.1.4 PROPERTIES OF ABSOLUTE VALUE. *If a and b are real numbers, then*

(a) $|-a| = |a|$ — A number and its negative have the same absolute value.

(b) $|ab| = |a|\,|b|$ — The absolute value of a product is the product of the absolute values.

(c) $|a/b| = |a|/|b|,\ b \neq 0$ — The absolute value of a ratio is the ratio of the absolute values.

(d) $|a + b| \leq |a| + |b|$ — The *triangle inequality*

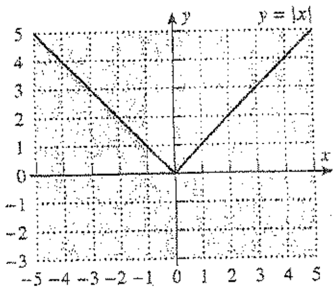


Figure 1.1.10

The graph of the function $f(x) = |x|$ can be obtained by graphing the two parts of the equation

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

separately. For $x \geq 0$, the graph of $y = x$ is a ray of slope 1 with its endpoint at the origin, and for $x < 0$, the graph of $y = -x$ is a ray of slope -1 with its endpoint at the origin. Combining the two parts produces the V-shaped graph in Figure 1.1.10.

Absolute values have important relationships to square roots. To see why this is so, recall from algebra that every positive real number x has two square roots, one positive and one negative. By definition, the symbol $\sqrt{x}$ denotes the *positive* square root of x. To denote the negative square root you must write $-\sqrt{x}$. For example, the positive square root of 9 is $\sqrt{9} = 3$, and the negative square root is $-\sqrt{9} = -3$. (Do not make the mistake of writing $\sqrt{9} = \pm 3$.)

Care must be exercised in simplifying expressions of the form $\sqrt{x^2}$, since it is *not* always true that $\sqrt{x^2} = x$. This equation is correct if x is nonnegative, but it is false for negative x. For example, if $x = -4$, then

$$\sqrt{x^2} = \sqrt{(-4)^2} = \sqrt{16} = 4 \neq x$$

A statement that is correct for all real values of x is

$$\sqrt{x^2} = |x| \qquad (2$$

TECHNOLOGY MASTERY

Verify (2) by using a graphing utility to show that the equations $y = \sqrt{x^2}$ and $y = |x|$ have the same graph.

FUNCTIONS DEFINED PIECEWISE

The absolute value function $f(x) = |x|$ is an example of a function that is defined *piecewis* in the sense that the formula for f changes, depending on the value of x.

▶ **Example 4** Sketch the graph of the function defined piecewise by the formula

$$f(x) = \begin{cases} 0, & x \leq -1 \\ \sqrt{1 - x^2}, & -1 < x < 1 \\ x, & x \geq 1 \end{cases}$$

Example of a Piecewise graph

Solution. The formula for f changes at the points $x = -1$ and $x = 1$. (We call these tl *breakpoints* for the formula.) A good procedure for graphing functions defined piecewi is to graph the function separately over the open intervals determined by the breakpoint and then graph f at the breakpoints themselves. For the function f in this example tl graph is the horizontal ray $y = 0$ on the interval $(-\infty, -1)$, it is the semicircle $y = \sqrt{1 -}$ on the interval $(-1, 1)$, and it is the ray $y = x$ on the interval $(1, +\infty)$. The formula for specifies that the equation $y = 0$ applies at the breakpoint -1 [so $y = f(-1) = 0$], and specifies that the equation $y = x$ applies at the breakpoint 1 [so $y = f(1) = 1$]. The gra of f is shown in Figure 1.1.11. ◀

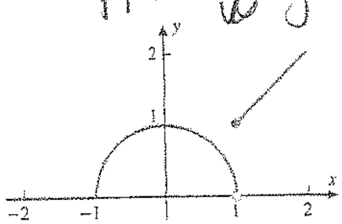


Figure 1.1.11

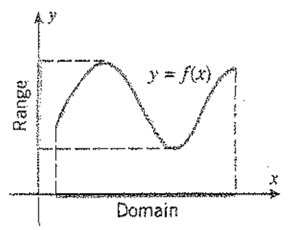


Figure 1.1.13 The projection of $y = f(x)$ on the x-axis is the set of allowable x-values for f, and the projection on the y-axis is the set of corresponding y-values.

The domain and range of a function f can be pictured by projecting the graph of $y = f(x)$ onto the coordinate axes as shown in Figure 1.1.13.

▶ Example 6 Find the natural domain of

(a) $f(x) = x^3$ (b) $f(x) = 1/[(x-1)(x-3)]$

(c) $f(x) = \tan x$ (d) $f(x) = \sqrt{x^2 - 5x + 6}$

Solution (a). The function f has real values for all real x, so its natural domain is the interval $(-\infty, +\infty)$.

Solution (b). The function f has real values for all real x, except $x = 1$ and $x = 3$, where divisions by zero occur. Thus, the natural domain is

$$\{x : x \neq 1 \text{ and } x \neq 3\} = (-\infty, 1) \cup (1, 3) \cup (3, +\infty)$$

Solution (c). Since $f(x) = \tan x = \sin x / \cos x$, the function f has real values except where $\cos x = 0$, and this occurs when x is an odd integer multiple of $\pi/2$. Thus, the natural domain consists of all real numbers except

$$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

For a review of trigonometry see Appendix A.

Solution (d). The function f has real values, except when the expression inside the radical is negative. Thus the natural domain consists of all real numbers x such that

$$x^2 - 5x + 6 = (x-3)(x-2) \geq 0$$

This inequality is satisfied if $x \leq 2$ or $x \geq 3$ (verify), so the natural domain of f is

$$(-\infty, 2] \cup [3, +\infty) \blacktriangleleft$$

In some cases we will include the domain explicitly when defining a function. For example, if $f(x) = x^2$ is the area of a square of side x, then we can write

$$f(x) = x^2, \quad x \geq 0$$

to indicate that we take the domain of f to be the set of nonnegative real numbers (Figure 1.1.14).

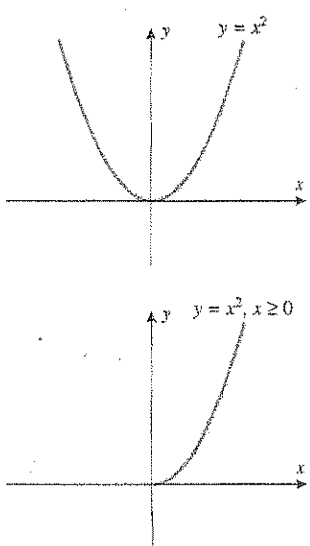


Figure 1.1.14

THE EFFECT OF ALGEBRAIC OPERATIONS ON THE DOMAIN

Algebraic expressions are frequently simplified by canceling common factors in the numerator and denominator. However, care must be exercised when simplifying formulas for functions in this way, since this process can alter the domain.

▶ Example 7 The natural domain of the function

$$f(x) = \frac{x^2 - 4}{x - 2} \tag{3}$$

consists of all real x except $x = 2$. However, if we factor the numerator and then cancel the common factor in the numerator and denominator, we obtain

$$f(x) = \frac{(x-2)(x+2)}{x-2} = x + 2 \tag{4}$$

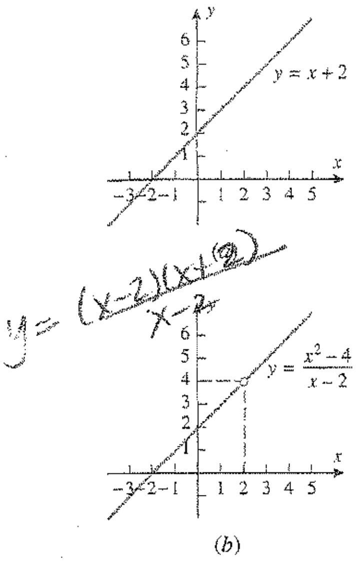


Figure 1.1.15

Since the right side of (4) has a value of $f(2) = 4$, whereas $f(2)$ was undefined in (3), the algebraic simplification has changed the function. Geometrically, the graph of (4) is the line in Figure 1.1.15*a*, whereas the graph of (3) is the same line but with a hole at $x = 2$, since the function is undefined there (Figure 1.1.15*b*). In short, the geometric effect of the algebraic cancellation is to eliminate the hole in the original graph. ◀

Sometimes alterations to the domain of a function that result from algebraic simplification are irrelevant to the problem at hand and can be ignored. However, if the domain must be preserved, then one must impose the restrictions on the simplified function explicitly. For example, if we wanted to preserve the domain of the function in Example 7, then we would have to express the simplified form of the function as

$$f(x) = x + 2, \quad x \neq 2$$

▶ **Example 8** Find the domain and range of

(a) $f(x) = 2 + \sqrt{x-1}$ (b) $f(x) = (x+1)/(x-1)$

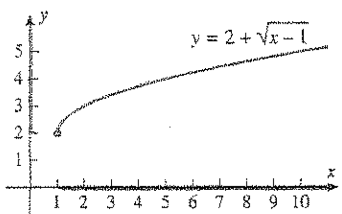


Figure 1.1.16

Solution (a). Since no domain is stated explicitly, the domain of f is the natural domain $[1, +\infty)$. As x varies over the interval $[1, +\infty)$, the value of $\sqrt{x-1}$ varies over the interval $[0, +\infty)$, so the value of $f(x) = 2 + \sqrt{x-1}$ varies over the interval $[2, +\infty)$, which is the range of f. The domain and range are highlighted in green on the x- and y-axes in Figure 1.1.16.

Solution (b). The given function f is defined for all real x, except $x = 1$, so the natural domain of f is

$$\{x : x \neq 1\} = (-\infty, 1) \cup (1, +\infty)$$

Set notation

To determine the range it will be convenient to introduce a dependent variable

$$y = \frac{x+1}{x-1} \tag{5}$$

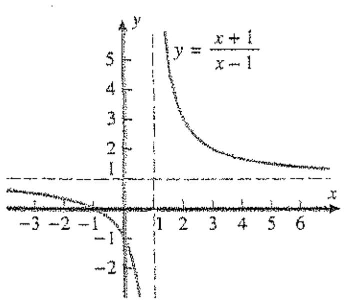


Figure 1.1.17

Although the set of possible y-values is not immediately evident from this equation, the graph of (5), which is shown in Figure 1.1.17, suggests that the range of f consists of all y, except $y = 1$. To see that this is so, we solve (5) for x in terms of y:

$$\begin{aligned}(x-1)y &= x+1\\ xy - y &= x+1\\ xy - x &= y+1\\ x(y-1) &= y+1\\ x &= \frac{y+1}{y-1}\end{aligned}$$

It is now evident from the right side of this equation that $y = 1$ is not in the range; otherwise we would have a division by zero. No other values of y are excluded by this equation, so the range of the function f is $\{y : y \neq 1\} = (-\infty, 1) \cup (1, +\infty)$, which agrees with the result obtained graphically. ◀

DOMAIN AND RANGE IN APPLIED PROBLEMS

In applications, physical considerations often impose restrictions on the domain and range of a function.

Example 9 An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides (Figure 1.1.18*a*).

(a) Let V be the volume of the box that results when the squares have sides of length x. Find a formula for V as a function of x.

(b) Find the domain of V.

(c) Use the graph of V given in Figure 1.1.18*c* to estimate the range of V.

(d) Describe in words what the graph tells you about the volume.

a graphing calculator is helpful for this example

Solution (a). As shown in Figure 1.1.18*b*, the resulting box has dimensions $16 - 2x$ by $30 - 2x$ by x, so the volume $V(x)$ is given by

$$V(x) = (16 - 2x)(30 - 2x)x = 480x - 92x^2 + 4x^3$$

Solution (b). The domain is the set of x-values and the range is the set of V-values. Because x is a length, it must be nonnegative, and because we cannot cut out squares whose sides are more than 8 in long (why?), the x-values in the domain must satisfy

$$0 \le x \le 8$$

Solution (c). From the graph of V versus x in Figure 1.1.18*c* we estimate that the V-values in the range satisfy

$$0 \le V \le 725$$

Note that this is an approximation. Later we will show how to find the range exactly.

Solution (d). The graph tells us that the box of maximum volume occurs for a value of x that is between 3 and 4 and that the maximum volume is approximately 725 in^3. Moreover, the volume decreases toward zero as x gets closer to 0 or 8. ◄

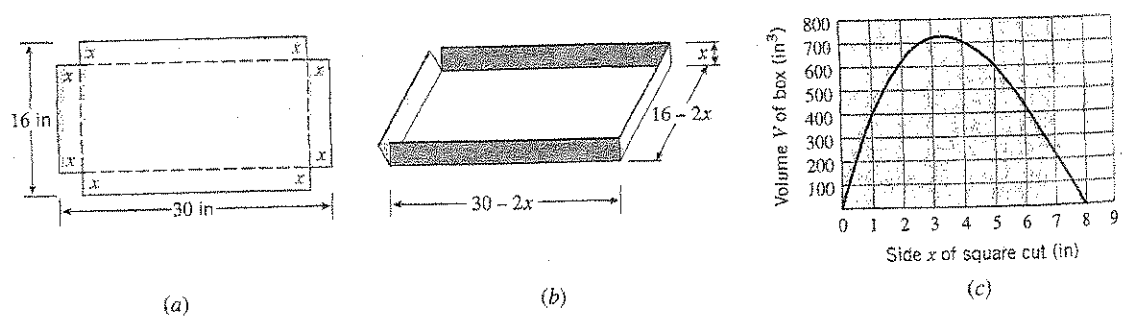


Figure 1.1.18

In applications involving time, formulas for functions are often expressed in terms of a variable t whose starting value is taken to be $t = 0$.

Example 10 At 8:05 A.M. a car is clocked at 100 ft/s by a radar detector that is positioned at the edge of a straight highway. Assuming that the car maintains a constant speed between 8:05 A.M. and 8:06 A.M., find a function $D(t)$ that expresses the distance traveled by the car during that time interval as a function of the time t.

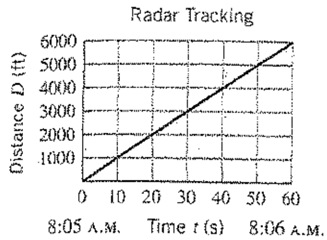


Figure 1.1.19

Solution. It would be clumsy to use clock time for the variable t, so let us agree to measure the elapsed time in seconds, starting with $t = 0$ at 8:05 A.M. and ending with $t = 60$ at 8:06 A.M. At each instant, the distance traveled (in ft) is equal to the speed of the car (in ft/s) multiplied by the elapsed time (in s). Thus,

$$D(t) = 100t, \quad 0 \le t \le 60$$

The graph of D versus t is shown in Figure 1.1.19. ◄

ISSUES OF SCALE AND UNITS

In geometric problems where you want to preserve the "true" shape of a graph, you must use units of equal length on both axes. For example, if you graph a circle in a coordinate system in which 1 unit in the y-direction is smaller than 1 unit in the x-direction, then the circle will be squashed vertically into an elliptical shape (Figure 1.1.20). You must also use units of equal length when you want to apply the distance formula

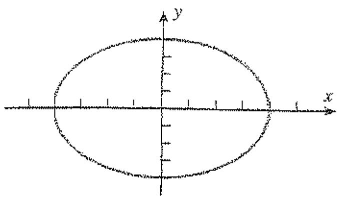
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

to calculate the distance between two points (x_1, y_1) and (x_2, y_2) in the xy-plane.

However, sometimes it is inconvenient or impossible to display a graph using units of equal length. For example, consider the equation

$$y = x^2$$

If we want to show the portion of the graph over the interval $-3 \le x \le 3$, then there is no problem using units of equal length, since y only varies from 0 to 9 over that interval. However, if we want to show the portion of the graph over the interval $-10 \le x \le 10$, then there is a problem keeping the units equal in length, since the value of y varies between 0 and 100. In this case the only reasonable way to show all of the graph that occurs over the interval $-10 \le x \le 10$ is to compress the unit of length along the y-axis, as illustrated in Figure 1.1.21.



The circle is squashed because 1 unit on the y-axis has a smaller length than 1 unit on the x-axis.

Figure 1.1.20

In applications where the variables on the two axes have unrelated units (say, centimeters on the y-axis and seconds on the x-axis), then nothing is gained by requiring the units to have equal lengths; choose the lengths to make the graph as clear as possible.

Section 1.1

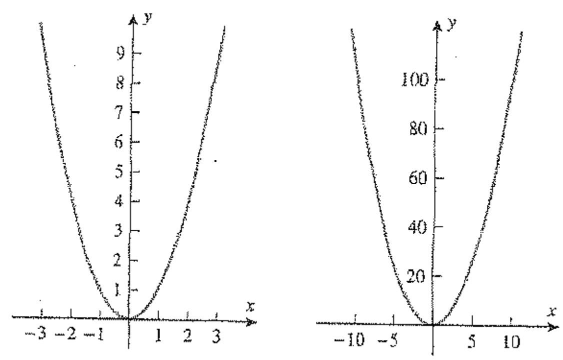


Figure 1.1.21

✔ QUICK CHECK EXERCISES 1.1 (See page 16 for answers.)

1. Let $f(x) = \sqrt{x+1} + 4$.
   (a) The natural domain of f is ______
   (b) $f(3) =$ ______
   (c) $f(t^2 - 1) =$ ______
   (d) $f(x) = 7$ if $x =$ ______
   (e) The range of f is ______.

2. Line segments in an xy-plane form "letters" as depicted.

   (a) If the y-axis is parallel to the letter I, which of the letters represent the graph of $y = f(x)$ for some function f?

Find the Domain Sec 1.1

12. (a) $f(x) = \dfrac{1}{5x+7}$ (b) $h(x) = \sqrt{x - 3x^2}$
(c) $G(x) = \sqrt{\dfrac{x^2-4}{x-4}}$ (d) $f(x) = \dfrac{x^2-1}{x+1}$
(e) $h(x) = \dfrac{3}{2 - \cos x}$

13. (a) $f(x) = \sqrt{3-x}$ (b) $g(x) = \sqrt{4-x^2}$
(c) $h(x) = 3 + \sqrt{x}$ (d) $G(x) = x^3 + 2$
(e) $H(x) = 3 \sin x$

14. (a) $f(x) = \sqrt{3x-2}$ (b) $g(x) = \sqrt{9-4x^2}$
(c) $h(x) = \dfrac{1}{3+\sqrt{x}}$ (d) $G(x) = \dfrac{3}{x}$
(e) $H(x) = \sin^2 \sqrt{x}$

FOCUS ON CONCEPTS

15. (a) If you had a device that could record the Earth's population continuously, would you expect the graph of population versus time to be a continuous (unbroken) curve? Explain what might cause breaks in the curve.
(b) Suppose that a hospital patient receives an injection of an antibiotic every 8 hours and that between injections the concentration C of the antibiotic in the bloodstream decreases as the antibiotic is absorbed by the tissues. What might the graph of C versus the elapsed time t look like?

16. (a) If you had a device that could record the temperature of a room continuously over a 24-hour period, would you expect the graph of temperature versus time to be a continuous (unbroken) curve? Explain your reasoning.
(b) If you had a computer that could track the number of boxes of cereal on the shelf of a market continuously over a 1-week period, would you expect the graph of the number of boxes on the shelf versus time to be a continuous (unbroken) curve? Explain your reasoning.

17. A boat is bobbing up and down on some gentle waves. Suddenly it gets hit by a large wave and sinks. Sketch a rough graph of the height of the boat above the ocean floor as a function of time.

18. A cup of hot coffee sits on a table. You pour in some cool milk and let it sit for an hour. Sketch a rough graph of the temperature of the coffee as a function of time.

19. Use the equation $y = x^2 - 6x + 8$ to answer the following questions.
(a) For what values of x is $y = 0$?
(b) For what values of x is $y = -10$?
(c) For what values of x is $y \geq 0$?
(d) Does y have a minimum value? A maximum value? If so, find them.

20. Use the equation $y = 1 + \sqrt{x}$ to answer the following questions.
(a) For what values of x is $y = 4$?
(b) For what values of x is $y = 0$?
(c) For what values of x is $y \geq 6$?
(d) Does y have a minimum value? A maximum value? If so, find them.

21. As shown in the accompanying figure, a pendulum of constant length L makes an angle θ with its vertical position. Express the height h as a function of the angle θ. TRIG!

22. Express the length L of a chord of a circle with radius 10 cm as a function of the central angle θ (see the accompanying figure). TRIG!

think Law of Cosines

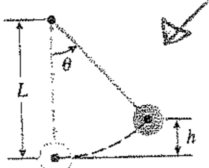


Figure Ex-21

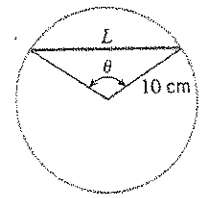


Figure Ex-22

23–24 Express the function in piecewise form without using absolute values. [*Suggestion:* It may help to generate the graph of the function.]

23. (a) $f(x) = |x| + 3x + 1$ (b) $g(x) = |x| + |x-1|$

24. (a) $f(x) = 3 + |2x - 5|$ (b) $g(x) = 3|x-2| - |x+1|$

25. As shown in the accompanying figure, an open box is to be constructed from a rectangular sheet of metal, 8 in by 15 in, by cutting out squares with sides of length x from each corner and bending up the sides.
Ex 9
(a) Express the volume V as a function of x.
(b) Find the domain of V.
(c) Plot the graph of the function V obtained in part (a) and estimate the range of this function.
(d) In words, describe how the volume V varies with x, and discuss how one might construct boxes of maximum volume.

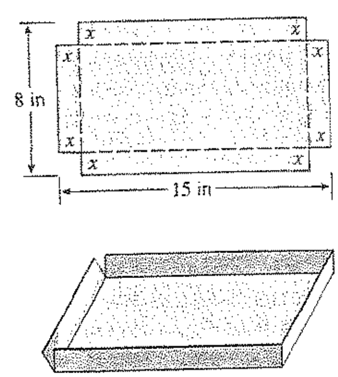


Figure Ex-25

26. Repeat Exercise 25 assuming the box is constructed in the same fashion from a 6-inch-square sheet of metal.

Sec 1.1

27. A construction company has adjoined a 1000-ft^2 rectangular enclosure to its office building. Three sides of the enclosure are fenced in. The side of the building adjacent to the enclosure is 100 ft long and a portion of this side is used as the fourth side of the enclosure. Let x and y be the dimensions of the enclosure, where x is measured parallel to the building, and let L be the length of fencing required for those dimensions.

(a) Find a formula for L in terms of x and y.
(b) Find a formula that expresses L as a function of x alone.
(c) What is the domain of the function in part (b)?
(d) Plot the function in part (b) and estimate the dimensions of the enclosure that minimize the amount of fencing required.

28. As shown in the accompanying figure, a camera is mounted at a point 3000 ft from the base of a rocket launching pad. The rocket rises vertically when launched, and the camera's elevation angle is continually adjusted to follow the bottom of the rocket.

(a) Express the height x as a function of the elevation angle θ.
(b) Find the domain of the function in part (a).
(c) Plot the graph of the function in part (a) and use it to estimate the height of the rocket when the elevation angle is $\pi/4 \approx 0.7854$ radian. Compare this estimate to the exact height. [*Suggestion:* If you are using a graphing calculator, the trace and zoom features will be helpful here.]

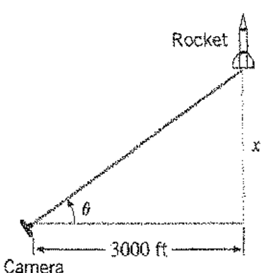


Figure Ex-28

29. A soup company wants to manufacture a can in the shape of a right circular cylinder that will hold 500 cm^3 of liquid. The material for the top and bottom costs 0.02 cent/cm^2, and the material for the sides costs 0.01 cent/cm^2.

(a) Estimate the radius r and the height h of the can that costs the least to manufacture. [*Suggestion:* Express the cost C in terms of r.]
(b) Suppose that the tops and bottoms of radius r are punched out from square sheets with sides of length $2r$ and the scraps are waste. If you allow for the cost of the waste, would you expect the can of least cost to be taller or shorter than the one in part (a)? Explain.
(c) Estimate the radius, height, and cost of the can in part (b), and determine whether your conjecture was correct.

30. The designer of a sports facility wants to put a quarter-mile (1320 ft) running track around a football field, oriented as in the accompanying figure. The football field is 360 ft long (including the end zones) and 160 ft wide. The track consists of two straightaways and two semicircles, with the straightaways extending at least the length of the football field.

(a) Show that it is possible to construct a quarter-mile track around the football field. [*Suggestion:* Find the shortest track that can be constructed around the field.]
(b) Let L be the length of a straightaway (in feet), and let x be the distance (in feet) between a sideline of the football field and a straightaway. Make a graph of L versus x.
(c) Use the graph to estimate the value of x that produces the shortest straightaways, and then find this value of x exactly.
(d) Use the graph to estimate the length of the longest possible straightaways, and then find that length exactly.

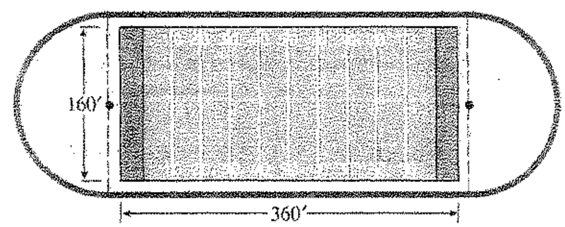


Figure Ex-30

31–32 (i) Explain why the function f has one or more holes in its graph, and state the x-values at which those holes occur. (ii) Find a function g whose graph is identical to that of f, but without the holes.

31. $f(x) = \dfrac{(x+2)(x^2-1)}{(x+2)(x-1)}$ **32.** $f(x) = \dfrac{x^2 + |x|}{|x|}$

33. In 2001 the National Weather Service introduced a new wind chill temperature (WCT) index. For a given outside temperature T and wind speed v, the wind chill temperature index is the equivalent temperature that exposed skin would feel with a wind speed of v mi/h. Based on a more accurate model of cooling due to wind, the new formula is

$$\text{WCT} = \begin{cases} T, & 0 \le v \le 3 \\ 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}, & 3 < v \end{cases}$$

where T is the temperature in °F, v is the wind speed in mi/h, and WCT is the equivalent temperature in °F. Find the WCT to the nearest degree if $T = 25$°F and
(a) $v = 3$ mi/h (b) $v = 15$ mi/h (c) $v = 46$ mi/h.

Source: Adapted from UMAP Module 658, *Windchill*, W. Bosch and L. Cobb, COMAP, Arlington, MA.

34–36 Use the formula for the wind chill temperature index described in Exercise 33.

34. Find the air temperature to the nearest degree if the WCT is reported as −60°F with a wind speed of 48 mi/h.

Omit 1.2 Sec 1.3

The thought process in this example suggests a general procedure for decomposing a function h into a composition $h = f \circ g$:

- Think about how you would evaluate $h(x)$ for a specific value of x, trying to break the evaluation into two steps performed in succession.
- The first operation in the evaluation will determine a function g and the second a function f.
- The formula for h can then be written as $h(x) = f(g(x))$.

For descriptive purposes, we will refer to g as the "inside function" and f as the "outside function" in the expression $f(g(x))$. The inside function performs the first operation and the outside function performs the second.

Example 5 Express $h(x) = (x-4)^5$ as a composition of two functions.

Solution. To evaluate $h(x)$ for a given value of x we would first compute $x-4$ and then raise the result to the fifth power. Therefore, the inside function (first operation) is

$$g(x) = x - 4$$

and the outside function (second operation) is

$$f(x) = x^5$$

so $h(x) = f(g(x))$. As a check,

$$f(g(x)) = [g(x)]^5 = (x-4)^5 = h(x) \blacktriangleleft$$

Example 6 Express $\sin(x^3)$ as a composition of two functions.

Solution. To evaluate $\sin(x^3)$, we would first compute x^3 and then take the sine, so $g(x) = x^3$ is the inside function and $f(x) = \sin x$ the outside function. Therefore,

$$\sin(x^3) = f(g(x)) \qquad \boxed{g(x) = x^3 \text{ and } f(x) = \sin x} \blacktriangleleft$$

Table 1.3.1 gives some more examples of decomposing functions into compositions.

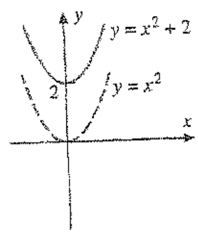
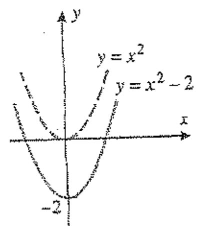
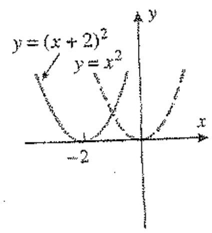
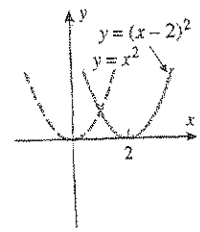
Table 1.3.1

FUNCTION	$g(x)$ INSIDE	$f(x)$ OUTSIDE	COMPOSITION
$(x^2+1)^{10}$	x^2+1	x^{10}	$(x^2+1)^{10} = f(g(x))$
$\sin^3 x$	$\sin x$	x^3	$\sin^3 x = f(g(x))$
$\tan(x^5)$	x^5	$\tan x$	$\tan(x^5) = f(g(x))$
$\sqrt{4-3x}$	$4-3x$	$\sqrt{x}$	$\sqrt{4-3x} = f(g(x))$
$8+\sqrt{x}$	$\sqrt{x}$	$8+x$	$8+\sqrt{x} = f(g(x))$
$\frac{1}{x+1}$	$x+1$	$\frac{1}{x}$	$\frac{1}{x+1} = f(g(x))$

Sec 1.3

You've seen this in TRIG!

Table 1.3.2

OPERATION ON $y = f(x)$	Add a positive constant c to $f(x)$	Subtract a positive constant c from $f(x)$	Add a positive constant c to x	Subtract a positive constant c from x
NEW EQUATION	$y = f(x) + c$	$y = f(x) - c$	$y = f(x + c)$	$y = f(x - c)$
GEOMETRIC EFFECT	Translates the graph of $y = f(x)$ up c units	Translates the graph of $y = f(x)$ down c units	Translates the graph of $y = f(x)$ left c units	Translates the graph of $y = f(x)$ right c units
EXAMPLE				

▸ **Example 8** Sketch the graph of

$$\text{(a)}\ y = \sqrt{x-3} \qquad \text{(b)}\ y = \sqrt{x+3}$$

Solution. The graph of the equation $y = \sqrt{x-3}$ can be obtained by translating the graph of $y = \sqrt{x}$ right 3 units, and the graph of $y = \sqrt{x+3}$ can be obtained by translating the graph of $y = \sqrt{x}$ left 3 units (Figure 1.3.3). ◂

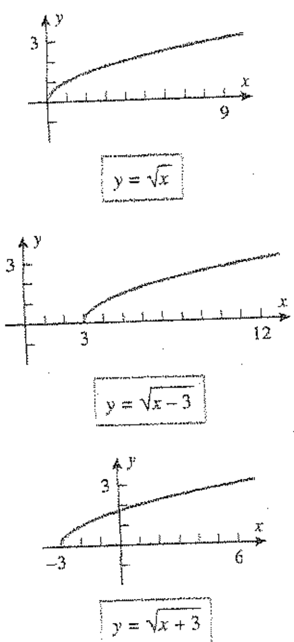


Figure 1.3.3

▸ **Example 9** Sketch the graph of $y = |x - 3| + 2$.

Solution. The graph can be obtained by two translations: first translate the graph of $y = |x|$ right 3 units to obtain the graph of $y = |x - 3|$, then translate this graph up 2 units to obtain the graph of $y = |x - 3| + 2$ (Figure 1.3.4). If desired, the same result can be obtained by performing the translations in the opposite order: first translate the graph of $|x|$ up 2 units to obtain the graph of $y = |x| + 2$, then translate this graph right 3 units to obtain the graph of $y = |x - 3| + 2$. ◂

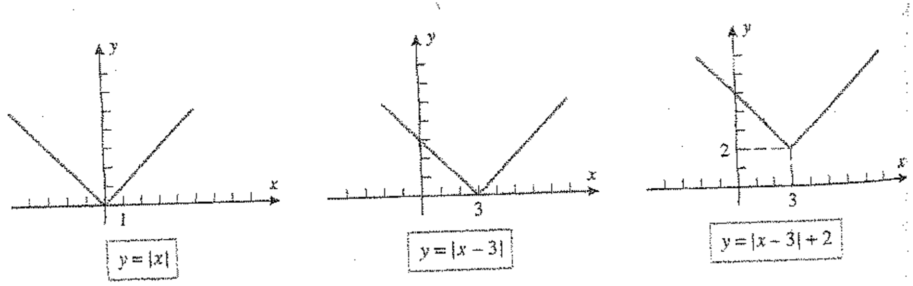
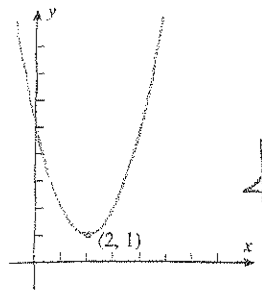


Figure 1.3.4

▸ **Example 10** Sketch the graph of $y = x^2 - 4x + 5$.

Solution. Completing the square on the first two terms yields

$$y = (x^2 - 4x + 4) - 4 + 5 = (x - 2)^2 + 1$$



$y = x^2 - 4x + 5$

Figure 1.3.5

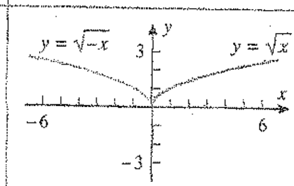
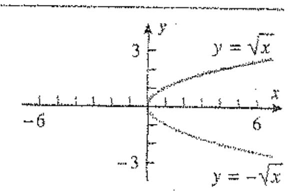
(see Web Appendix G for a review of this technique). In this form we see that the graph can be obtained by translating the graph of $y = x^2$ right 2 units because of the $x - 2$, and up 1 unit because of the $+1$ (Figure 1.3.5). ◀

REFLECTIONS

The graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ about the y-axis because the point (x, y) on the graph of $f(x)$ is replaced by $(-x, y)$. Similarly, the graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ about the x-axis because the point (x, y) on the graph of $f(x)$ is replaced by $(x, -y)$ [the equation $y = -f(x)$ is equivalent to $-y = f(x)$]. This is summarized in Table 1.3.3.

Symmetry about the x-axis & y-axis done in Alg!

Table 1.3.3

OPERATION ON $y = f(x)$	Replace x by $-x$	Multiply $f(x)$ by -1
NEW EQUATION	$y = f(-x)$	$y = -f(x)$
GEOMETRIC EFFECT	Reflects the graph of $y = f(x)$ about the y-axis	Reflects the graph of $y = f(x)$ about the x-axis
EXAMPLE		

▶ **Example 11** Sketch the graph of $y = \sqrt[3]{2 - x}$.

Solution. The graph can be obtained by a reflection and a translation: first reflect the graph of $y = \sqrt[3]{x}$ about the y-axis to obtain the graph of $y = \sqrt[3]{-x}$, then translate this graph right 2 units to obtain the graph of the equation $y = \sqrt[3]{-(x-2)} = \sqrt[3]{2-x}$ (Figure 1.3.6). ◀

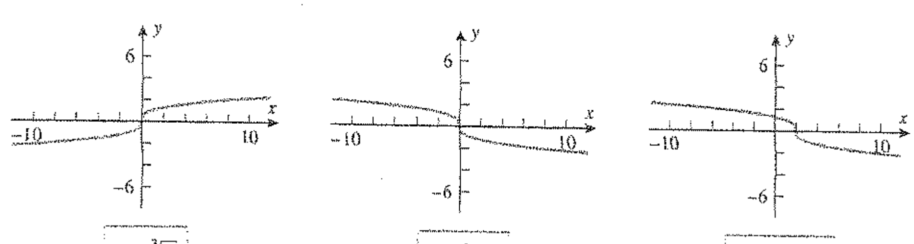


Figure 1.3.6

▶ **Example 12** Sketch the graph of $y = 4 - |x - 2|$.

Solution. The graph can be obtained by a reflection and two translations: first translate the graph of $y = |x|$ right 2 units to obtain the graph of $y = |x - 2|$; then reflect this graph about the x-axis to obtain the graph of $y = -|x - 2|$; and then translate this graph up 4 units to obtain the graph of the equation $y = -|x - 2| + 4 = 4 - |x - 2|$ (Figure 1.3.7). ◀

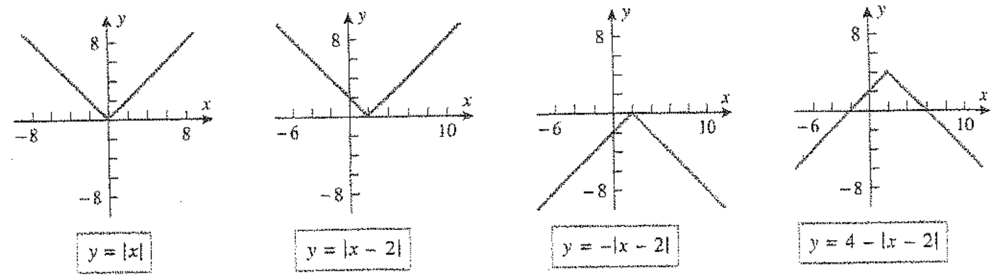


Figure 1.3.7

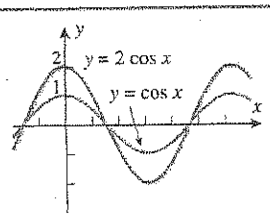
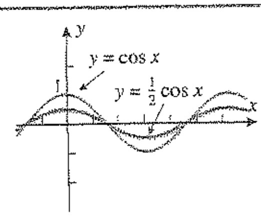
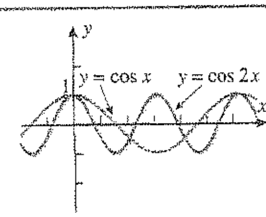
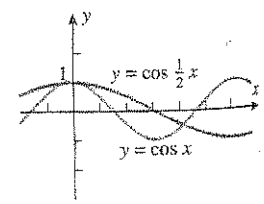
STRETCHES AND COMPRESSIONS

done in TRIG!

Multiplying $f(x)$ by a *positive* constant c has the geometric effect of stretching the graph of $y = f(x)$ in the y-direction by a factor of c if $c > 1$ and compressing it in the y-direction by a factor of $1/c$ if $0 < c < 1$. For example, multiplying $f(x)$ by 2 doubles each y-coordinate, thereby stretching the graph vertically by a factor of 2, and multiplying by $\frac{1}{2}$ cuts each y-coordinate in half, thereby compressing the graph vertically by a factor of 2. Similarly, multiplying x by a *positive* constant c has the geometric effect of compressing the graph of $y = f(x)$ by a factor of c in the x-direction if $c > 1$ and stretching it by a factor of $1/c$ if $0 < c < 1$. [If this seems backwards to you, then think of it this way: The value of $2x$ changes twice as fast as x, so a point moving along the x-axis from the origin will only have to move half as far for $y = f(2x)$ to have the same value as $y = f(x)$, thereby creating a horizontal compression of the graph.] All of this is summarized in Table 1.3.4.

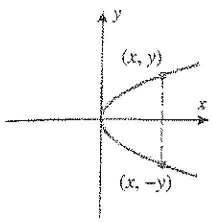
Describe the geometric effect of multiplying a function f by a *negative* constant in terms of reflection and stretching or compressing. What is the geometric effect of multiplying the independent variable of a function f by a *negative* constant?

Table 1.3.4

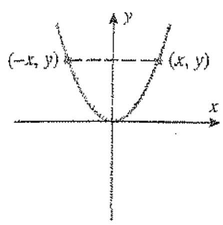
OPERATION ON $y = f(x)$	Multiply $f(x)$ by c ($c > 1$)	Multiply $f(x)$ by c ($0 < c < 1$)	Multiply x by c ($c > 1$)	Multiply x by c ($0 < c < 1$)
NEW EQUATION	$y = cf(x)$	$y = cf(x)$	$y = f(cx)$	$y = f(cx)$
GEOMETRIC EFFECT	Stretches the graph of $y = f(x)$ vertically by a factor of c	Compresses the graph of $y = f(x)$ vertically by a factor of $1/c$	Compresses the graph of $y = f(x)$ horizontally by a factor of c	Stretches the graph of $y = f(x)$ horizontally by a factor of $1/c$
EXAMPLE				

SYMMETRY

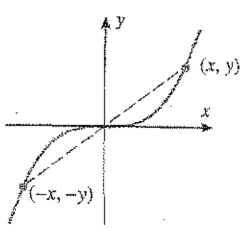
Figure 1.3.8 illustrates three types of symmetries: ***symmetry about the x-axis***, ***symmetry about the y-axis***, and ***symmetry about the origin***. As illustrated in the figure, a curve is symmetric about the x-axis if for each point (x, y) on the graph the point $(x, -y)$ is also on the graph, and it is symmetric about the y-axis if for each point (x, y) on the graph the point $(-x, y)$ is also on the graph. A curve is symmetric about the origin if for each point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. (Equivalently, a graph is symmetric about the origin if rotating the graph 180° about the origin leaves it unchanged.) This suggests the following symmetry tests.



Symmetric about the x-axis



Symmetric about the y-axis



Symmetric about the origin

Figure 1.3.8

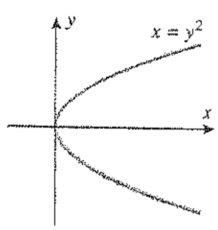


Figure 1.3.9

1.3.3 THEOREM (*Symmetry Tests*).

(*a*) *A plane curve is symmetric about the y-axis if and only if replacing x by −x in its equation produces an equivalent equation.*

(*b*) *A plane curve is symmetric about the x-axis if and only if replacing y by −y in its equation produces an equivalent equation.*

(*c*) *A plane curve is symmetric about the origin if and only if replacing both x by −x and y by −y in its equation produces an equivalent equation.*

▶ **Example 13** Use Theorem 1.3.3 to identify symmetries in the graph of $x = y^2$.

Solution. Replacing y by $-y$ yields $x = (-y)^2$, which simplifies to the original equation $x = y^2$. Thus, the graph is symmetric about the x-axis. The graph is not symmetric about the y-axis because replacing x by $-x$ yields $-x = y^2$, which is not equivalent to the original equation $x = y^2$. Similarly, the graph is not symmetric about the origin because replacing x by $-x$ and y by $-y$ yields $-x = (-y)^2$, which simplifies to $-x = y^2$, and this is again not equivalent to the original equation. These results are consistent with the graph of $x = y^2$ shown in Figure 1.3.9. ◀

EVEN AND ODD FUNCTIONS

DONE IN TRIG + AG!

A function f is said to be an ***even function*** if

$$f(-x) = f(x) \tag{8}$$

and is said to be an ***odd function*** if

$$f(-x) = -f(x) \tag{9}$$

Geometrically, the graphs of even functions are symmetric about the y-axis because replacing x by $-x$ in the equation $y = f(x)$ yields $y = f(-x)$, which is equivalent to the original equation $y = f(x)$ by (8) (see Figure 1.3.10). Similarly, it follows from (9) that graphs of odd functions are symmetric about the origin (see Figure 1.3.11). Some examples of even functions are x^2, x^4, x^6, and $\cos x$; and some examples of odd functions are x^3, x^5, x^7, and $\sin x$.

Explain why the graph of a nonzero function cannot be symmetric about the x-axis.

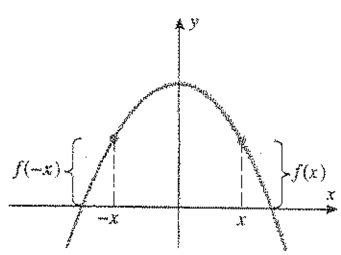


Figure 1.3.10 This is the graph of an even function since $f(-x) = f(x)$.

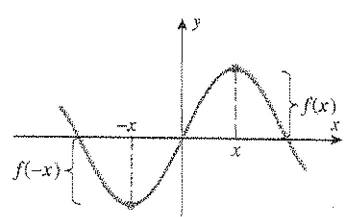


Figure 1.3.11 This is the graph of an odd function since $f(-x) = -f(x)$.

QUICK CHECK EXERCISES 1.3 (See page 39 for answers.)

1. Let $f(x) = 3\sqrt{x} - 2$ and $g(x) = |x|$. In each part, give the formula for the function and state the corresponding domain.
   (a) $f + g$: ________ Domain: ________
   (b) $f - g$: ________ Domain: ________
   (c) fg: ________ Domain: ________
   (d) f/g: ________ Domain: ________

2. Let $f(x) = 2 - x^2$ and $g(x) = \sqrt{x}$. In each part, give the formula for the composition and state the corresponding domain.
   (a) $f \circ g$: ________ Domain: ________
   (b) $g \circ f$: ________ Domain: ________

3. The graph of $y = 1 + (x - 2)^2$ may be obtained by shifting the graph of $y = x^2$ ________ (left/right) by ________ unit(s) and then shifting this new graph ________ (up/down) by ________ unit(s).

4. Let
$$f(x) = \begin{cases} |x+1|, & -2 \le x \le 0 \\ |x-1|, & 0 < x \le 2 \end{cases}$$
   (a) The letter of the alphabet that most resembles the graph of f is ________.
   (b) Is f an even function?

5-19 Describe the shifts, stretches/compresses

EXERCISE SET 1.3 Graphing Utility

FOCUS ON CONCEPTS

1. The graph of a function f is shown in the accompanying figure. Sketch the graphs of the following equations.
   (a) $y = f(x) - 1$ (b) $y = f(x - 1)$
   (c) $y = \frac{1}{2}f(x)$ (d) $y = f\left(-\frac{1}{2}x\right)$

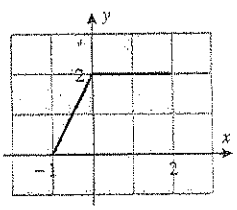


Figure Ex-1

2. Use the graph in Exercise 1 to sketch the graphs of the following equations.
   (a) $y = -f(-x)$ (b) $y = f(2 - x)$
   (c) $y = 1 - f(2 - x)$ (d) $y = \frac{1}{2}f(2x)$

3. The graph of a function f is shown in the accompanying figure. Sketch the graphs of the following equations.
   (a) $y = f(x + 1)$ (b) $y = f(2x)$
   (c) $y = |f(x)|$ (d) $y = 1 - |f(x)|$

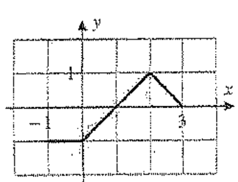


Figure Ex-3

4. Use the graph in Exercise 3 to sketch the graph of the equation $y = f(|x|)$.

5–10 Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = x^2$ appropriately, and then use a graphing utility to confirm that your sketch is correct.

5. $y = -2(x + 1)^2 - 3$
6. $y = \frac{1}{2}(x - 3)^2 + 2$
7. $y = x^2 + 6x$
8. $y = x^2 + 6x - 10$
9. $y = 1 + 2x - x^2$
10. $y = \frac{1}{2}(x^2 - 2x + 3)$

11–14 Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = \sqrt{x}$ appropriately, and then use a graphing utility to confirm that your sketch is correct.

11. $y = 3 - \sqrt{x + 1}$
12. $y = 1 + \sqrt{x - 4}$
13. $y = \frac{1}{2}\sqrt{x} + 1$
14. $y = -\sqrt{3x}$

15–18 Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = 1/x$ appropriately, and then use a graphing utility to confirm that your sketch is correct.

15. $y = \dfrac{1}{x - 3}$
16. $y = \dfrac{1}{1 - x}$
17. $y = 2 - \dfrac{1}{x + 1}$
18. $y = \dfrac{x - 1}{x}$

19–22 Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = |x|$ appropriately, and then use a graphing utility to confirm that your sketch is correct.

19. $y = |x + 2| - 2$
20. $y = 1 - |x - 3|$

Sec 1.3 cont'd

21. $y = |2x - 1| + 1$

22. $y = \sqrt{x^2 - 4x + 4}$

23–26 Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of $y = \sqrt[3]{x}$ appropriately, and then use a graphing utility to confirm that your sketch is correct.

23. $y = 1 - 2\sqrt[3]{x}$

24. $y = \sqrt[3]{x - 2} - 3$

25. $y = 2 + \sqrt[3]{x + 1}$

26. $y + \sqrt[3]{x - 2} = 0$

27. (a) Sketch the graph of $y = x + |x|$ by adding the corresponding y-coordinates on the graphs of $y = x$ and $y = |x|$.
(b) Express the equation $y = x + |x|$ in piecewise form with no absolute values, and confirm that the graph you obtained in part (a) is consistent with this equation.

28. Sketch the graph of $y = x + (1/x)$ by adding corresponding y-coordinates on the graphs of $y = x$ and $y = 1/x$. Use a graphing utility to confirm that your sketch is correct.

29–30 Find formulas for $f + g$, $f - g$, fg, and f/g, and state the domains of the functions.

29. $f(x) = 2\sqrt{x - 1},\ g(x) = \sqrt{x - 1}$

30. $f(x) = \dfrac{x}{1 + x^2},\ g(x) = \dfrac{1}{x}$

31. Let $f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$. Find
(a) $f(g(2))$ (b) $g(f(4))$
(c) $f(f(16))$ (d) $g(g(0))$.

32. Let $g(x) = \pi - x^2$ and $h(x) = \cos x$. Find
(a) $g(h(0))$ (b) $h(g(\sqrt{\pi/2}))$
(c) $g(g(1))$ (d) $h(h(\pi/2))$.

33. Let $f(x) = x^2 + 1$. Find
(a) $f(t^2)$ (b) $f(t + 2)$ (c) $f(x + 2)$
(d) $f\left(\dfrac{1}{x}\right)$ (e) $f(x + h)$ (f) $f(-x)$
(g) $f(\sqrt{x})$ (h) $f(3x)$.

34. Let $g(x) = \sqrt{x}$. Find
(a) $g(5s + 2)$ (b) $g(\sqrt{x} + 2)$ (c) $3g(5x)$
(d) $\dfrac{1}{g(x)}$ (e) $g(g(x))$ (f) $(g(x))^2 - g(x^2)$
(g) $g(1/\sqrt{x})$ (h) $g((x - 1)^2)$.

35–38 Find formulas for $f \circ g$ and $g \circ f$, and state the domains of the compositions.

35. $f(x) = x^2,\ g(x) = \sqrt{1 - x}$

36. $f(x) = \sqrt{x - 3},\ g(x) = \sqrt{x^2 + 3}$

37. $f(x) = \dfrac{1 + x}{1 - x},\ g(x) = \dfrac{x}{1 - x}$

38. $f(x) = \dfrac{x}{1 + x^2},\ g(x) = \dfrac{1}{x}$

39–40 Find a formula for $f \circ g \circ h$.

39. $f(x) = x^2 + 1,\ g(x) = \dfrac{1}{x},\ h(x) = x^3$

40. $f(x) = \dfrac{1}{1 + x},\ g(x) = \sqrt[3]{x},\ h(x) = \dfrac{1}{x^3}$

41–44 Express f as a composition of two functions; that is, find g and h such that $f = g \circ h$. [*Note:* Each exercise has more than one solution.]

41. (a) $f(x) = \sqrt{x + 2}$ (b) $f(x) = |x^2 - 3x + 5|$

42. (a) $f(x) = x^2 + 1$ (b) $f(x) = \dfrac{1}{x - 3}$

43. (a) $f(x) = \sin^2 x$ (b) $f(x) = \dfrac{3}{5 + \cos x}$

44. (a) $f(x) = 3\sin(x^2)$ (b) $f(x) = 3\sin^2 x + 4\sin x$

45–46 Express F as a composition of three functions; that is, find f, g, and h such that $F = f \circ g \circ h$. [*Note:* Each exercise has more than one solution.]

45. (a) $F(x) = \left(1 + \sin(x^2)\right)^3$ (b) $F(x) = \sqrt{1 - \sqrt[3]{x}}$

46. (a) $F(x) = \dfrac{1}{1 - x^2}$ (b) $F(x) = |5 + 2x|$

FOCUS ON CONCEPTS

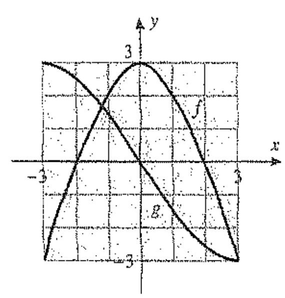
47. Use the data in the accompanying table to make a plot of $y = f(g(x))$.

x	−3	−2	−1	0	1	2	3
$f(x)$	−4	−3	−2	−1	0	1	2
$g(x)$	−1	0	1	2	3	−2	−3

Table Ex-47

48. Find the domain of $g \circ f$ for the functions f and g in Exercise 47.

49. Sketch the graph of $y = f(g(x))$ for the functions graphed in the accompanying figure.



Hint: f is even g is odd

Figure Ex-49

Sec 1.3 cont'd

50. Sketch the graph of $y = g(f(x))$ for the functions graphed in Exercise 49.

51. Use the graphs of f and g in Exercise 49 to estimate the solutions of the equations $f(g(x)) = 0$ and $g(f(x)) = 0$.

52. Use the table given in Exercise 47 to solve the equations $f(g(x)) = 0$ and $g(f(x)) = 0$.

53–56 Find

$$\frac{f(w) - f(x)}{w - x} \quad \text{and} \quad \frac{f(x+h) - f(x)}{h}$$

Simplify as much as possible.

53. $f(x) = 3x^2 - 5$ **54.** $f(x) = x^2 + 6x$

55. $f(x) = 1/x$ **56.** $f(x) = 1/x^2$

57. Classify the functions whose values are given in the accompanying table as even, odd, or neither.

x	−3	−2	−1	0	1	2	3
$f(x)$	5	3	2	3	1	−3	5
$g(x)$	4	1	−2	0	2	−1	−4
$h(x)$	2	−5	8	−2	8	−5	2

Table Ex-57

58. Complete the accompanying table so that the graph of $y = f(x)$ is symmetric about
(a) the y-axis (b) the origin.

x	−3	−2	−1	0	1	2	3
$f(x)$	1		−1	0		−5	

Table Ex-58

59. The accompanying figure shows a portion of a graph. Complete the graph so that the entire graph is symmetric about
(a) the x-axis (b) the y-axis (c) the origin.

60. The accompanying figure shows a portion of the graph of a function f. Complete the graph assuming that
(a) f is an even function (b) f is an odd function.

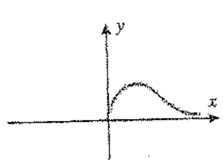


Figure Ex-59

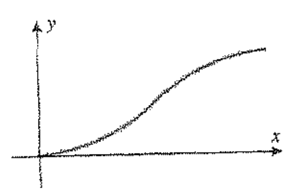


Figure Ex-60

61. Classify the functions graphed in the accompanying figure as even, odd, or neither.

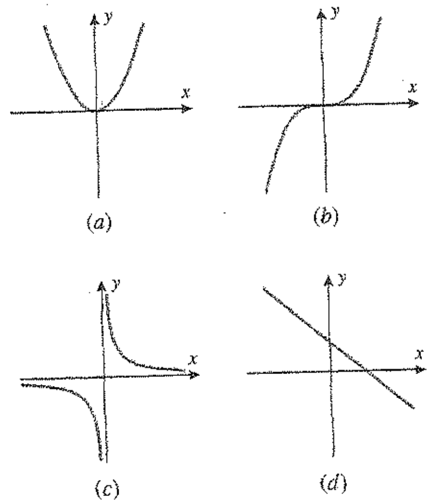


Figure Ex-61

62. In each part of the accompanying figure determine whether the graph is symmetric about the x-axis, the y-axis, the origin, or none of the preceding.

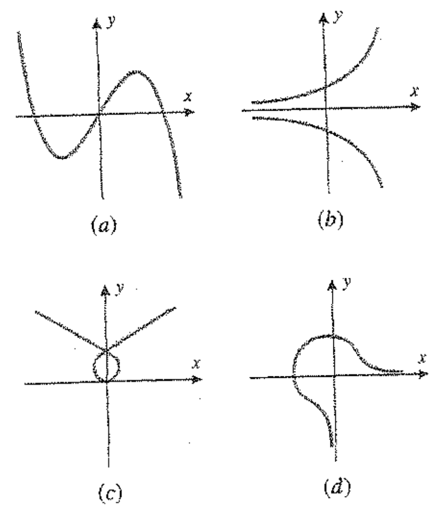


Figure Ex-62

63. In each part, classify the function as even, odd, or neither.
(a) $f(x) = x^2$ (b) $f(x) = x^3$
(c) $f(x) = |x|$ (d) $f(x) = x + 1$
(e) $f(x) = \dfrac{x^5 - x}{1 + x^2}$ (f) $f(x) = 2$

64. Suppose that the function f has domain all real numbers. Determine whether each function can be classified as even or odd. Explain.
(a) $g(x) = \dfrac{f(x) + f(-x)}{2}$ (b) $h(x) = \dfrac{f(x) - f(-x)}{2}$

65. Suppose that the function f has domain all real numbers. Show that f can be written as the sum of an even function and an odd function. [*Hint:* See Exercise 64.]

Sec 1.4

25. According to ***Coulomb's law***, the force F of attraction between positive and negative point charges is inversely proportional to the square of the distance x between them.

(a) Assuming that the force of attraction between two point charges is 0.0005 newton when the distance between them is 0.3 meter, find the constant of proportionality (with proper units).

(b) Find the force of attraction between the point charges when they are 3 meters apart.

(c) Make a graph of force versus distance for the two charges.

(d) What happens to the force as the particles get closer and closer together? What happens as they get farther and farther apart?

26. It follows from Newton's Law of Universal Gravitation that the weight W of an object (relative to the Earth) is inversely proportional to the square of the distance x between the object and the center of the Earth, that is, $W = C/x^2$.

(a) Assuming that a weather satellite weighs 2000 pounds on the surface of the Earth and that the Earth is a sphere of radius 4000 miles, find the constant C.

(b) Find the weight of the satellite when it is 1000 miles above the surface of the Earth.

(c) Make a graph of the satellite's weight versus its distance from the center of the Earth.

(d) Is there any distance from the center of the Earth at which the weight of the satellite is zero? Explain your reasoning.

FOCUS ON CONCEPTS

27. In each part, match the equation with one of the accompanying graphs, and give the equations for the horizontal and vertical asymptotes.

(a) $y = \dfrac{x^2}{x^2 - x - 2}$ (b) $y = \dfrac{x - 1}{x^2 - x - 6}$

(c) $y = \dfrac{2x^4}{x^4 + 1}$ (d) $y = \dfrac{4}{(x + 2)^2}$

AG Review

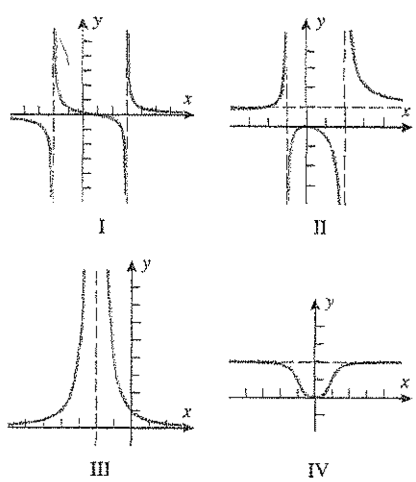


Figure Ex-27

28. Find an equation of the form $y = k/(x^2 + bx + c)$ whose graph is a reasonable match to that in the accompanying figure. Check your work with a graphing utility.

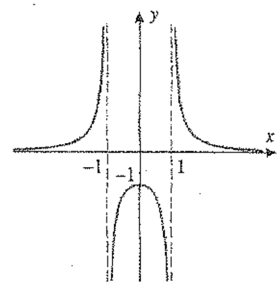


Figure Ex-28

29–30 Find an equation of the form $y = D + A \sin Bx$ or $y = D + A \cos Bx$ for each graph.

29.

TRIG

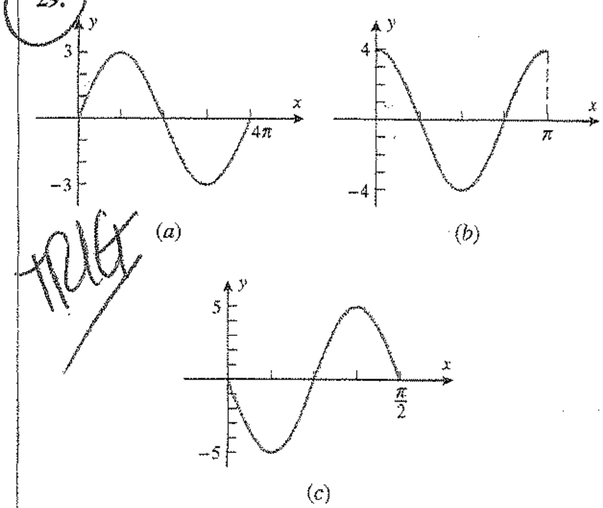


Figure Ex-29

30.

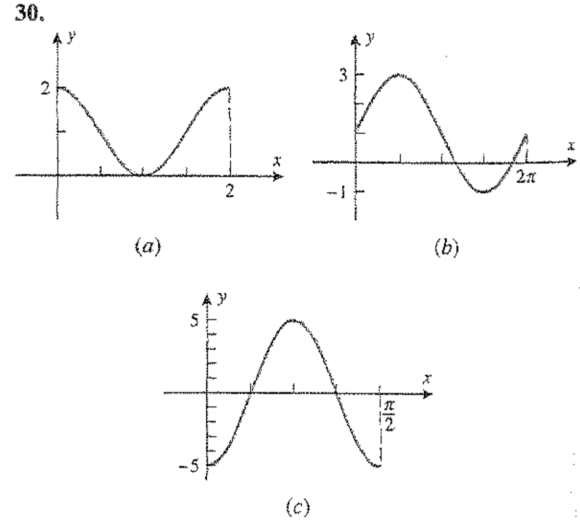


Figure Ex-30

Sec 1.5 Inverse functions

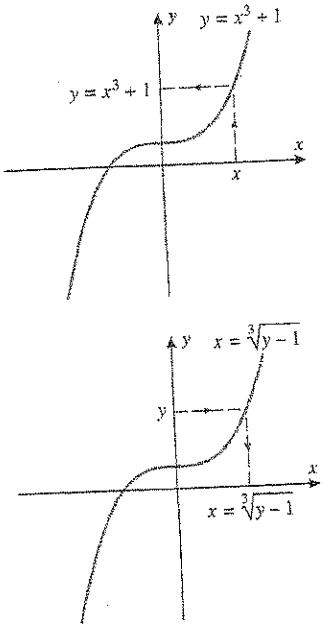


Figure 1.5.1

can be solved for x as a function of y:

$$x = \sqrt[3]{y-1} \qquad \boxed{x = g(y)}$$

The first equation is better for computing y if x is known, and the second is better for computing x if y is known (Figure 1.5.1).

Our primary interest in this section is to identify relationships that may exist between the functions f and g when an equation $y = f(x)$ is expressed as $x = g(y)$, or conversely. For example, consider the functions $f(x) = x^3 + 1$ and $g(y) = \sqrt[3]{y-1}$ discussed above. When these functions are composed in either order they cancel out the effect of one another in the sense that

$$\begin{aligned} g(f(x)) &= \sqrt[3]{f(x)-1} = \sqrt[3]{(x^3+1)-1} = x \\ f(g(y)) &= [g(y)]^3 + 1 = (\sqrt[3]{y-1})^3 + 1 = y \end{aligned} \tag{1}$$

Pairs of functions with these two properties are so important that there is some terminology for them.

1.5.1 DEFINITION. If the functions f and g satisfy the two conditions

$$g(f(x)) = x \text{ for every } x \text{ in the domain of } f$$
$$f(g(y)) = y \text{ for every } y \text{ in the domain of } g$$

then we say that ***f is an inverse of g*** and ***g is an inverse of f*** or that ***f and g are inverse functions***.

WARNING

If f is a function, then the -1 in the symbol f^{-1} always denotes an inverse and *never* an exponent. That is,

$$f^{-1}(x) \textit{ never means } \frac{1}{f(x)}$$

It can be shown (Exercise 34) that if a function f has an inverse, then that inverse is unique. Thus, if a function f has an inverse, then we are entitled to talk about "the" inverse of f, in which case we denote it by the symbol f^{-1}.

Example 1 The computations in (1) show that $g(y) = \sqrt[3]{y-1}$ is the inverse of $f(x) = x^3 + 1$. Thus, we can express g in inverse notation as

$$f^{-1}(y) = \sqrt[3]{y-1}$$

and we can express the equations in Definition 1.5.1 as

$$\begin{aligned} f^{-1}(f(x)) &= x \quad \text{for every } x \text{ in the domain of } f \\ f(f^{-1}(y)) &= y \quad \text{for every } y \text{ in the domain of } f^{-1} \end{aligned} \tag{2}$$

We will call these the ***cancellation equations*** for f and f^{-1}. ◀

CHANGING THE INDEPENDENT VARIABLE

The formulas in (2) use x as the independent variable for f and y as the independent variable for f^{-1}. Although it is often convenient to use different independent variables for f and f^{-1}, there will be occasions on which it is desirable to use the same independent variable for both. For example, if we want to graph the functions f and f^{-1} together in the same xy-coordinate system, then we would want to use x as the independent variable and y as the dependent variable for both functions. Thus, to graph the functions $f(x) = x^3 + 1$ and $f^{-1}(y) = \sqrt[3]{y-1}$ of Example 1 in the same xy-coordinate system, we would change the independent variable y to x, use y as the dependent variable for both functions, and graph the equations

$$y = x^3 + 1 \quad \text{and} \quad y = \sqrt[3]{x-1}$$

We will talk more about graphs of inverse functions later in this section, but for reference we give the following reformulation of the cancellation equations in (2) using x as the independent variable for both f and f^{-1}:

$$\begin{aligned} f^{-1}(f(x)) &= x \quad \text{for every } x \text{ in the domain of } f \\ f(f^{-1}(x)) &= x \quad \text{for every } x \text{ in the domain of } f^{-1} \end{aligned} \tag{3}$$

► **Example 2** Confirm each of the following.

(a) The inverse of $f(x) = 2x$ is $f^{-1}(x) = \frac{1}{2}x$.

(b) The inverse of $f(x) = x^3$ is $f^{-1}(x) = x^{1/3}$.

Solution (a).

$$f^{-1}(f(x)) = f^{-1}(2x) = \tfrac{1}{2}(2x) = x$$
$$f(f^{-1}(x)) = f\left(\tfrac{1}{2}x\right) = 2\left(\tfrac{1}{2}x\right) = x$$

The results in Example 2 should make sense to you intuitively, since the operations of multiplying by 2 and multiplying by $\frac{1}{2}$ in either order cancel the effect of one another, as do the operations of cubing and taking a cube root.

Solution (b).

$$f^{-1}(f(x)) = f^{-1}(x^3) = \left(x^3\right)^{1/3} = x$$
$$f(f^{-1}(x)) = f(x^{1/3}) = \left(x^{1/3}\right)^3 = x \blacktriangleleft$$

► **Example 3** Given that the function f has an inverse and that $f(3) = 5$, find $f^{-1}(5)$.

Solution. Apply f^{-1} to both sides of the equation $f(3) = 5$ to obtain

$$f^{-1}(f(3)) = f^{-1}(5)$$

and now apply the first equation in (3) to conclude that $f^{-1}(5) = 3$. ◄

In general, if f has an inverse and $f(a) = b$, then the procedure in Example 3 shows that $a = f^{-1}(b)$; that is, f^{-1} maps each output of f back into the corresponding input (Figure 1.5.2).

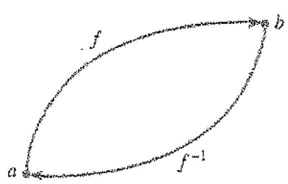


Figure 1.5.2 If f maps a to b, then f^{-1} maps b back to a.

DOMAIN AND RANGE OF INVERSE FUNCTIONS

The equations in (3) imply the following relationships between the domains and ranges of f and f^{-1}:

$$\begin{aligned} \text{domain of } f^{-1} &= \text{range of } f \\ \text{range of } f^{-1} &= \text{domain of } f \end{aligned} \tag{4}$$

One way to show that two sets are the same is to show that each is a subset of the other. Thus we can establish the first equality in (4) by showing that the domain of f^{-1} is a subset of the range of f and that the range of f is a subset of the domain of f^{-1}. We do this as follows: The first equation in (3) implies that f^{-1} is defined at $f(x)$ for all values of x in the domain of f, and this implies that the range of f is a subset of the domain of f^{-1}. Conversely, if x is in the domain of f^{-1}, then the second equation in (3) implies that x is in the range of f because it is the image of $f^{-1}(x)$. Thus, the domain of f^{-1} is a subset of the range of f. We leave the proof of the second equation in (4) as an exercise.

A METHOD FOR FINDING INVERSE FUNCTIONS

At the beginning of this section we observed that solving $y = f(x) = x^3 + 1$ for x as a function of y produces $x = f^{-1}(y) = \sqrt[3]{y-1}$. The following theorem shows that this is not accidental.

1.5.2 THEOREM. *If an equation $y = f(x)$ can be solved for x as a function of y, say $x = g(y)$, then f has an inverse and that inverse is $g(y) = f^{-1}(y)$.*

PROOF. Substituting $y = f(x)$ into $x = g(y)$ yields $x = g(f(x))$, which confirms the first equation in Definition 1.5.1, and substituting $x = g(y)$ into $y = f(x)$ yields $y = f(g(y))$, which confirms the second equation in Definition 1.5.1. ■

Theorem 1.5.2 provides us with the following procedure for finding the inverse of a function.

A Procedure for Finding the Inverse of a Function f

Step 1. Write down the equation $y = f(x)$.

Step 2. If possible, solve this equation for x as a function of y.

Step 3. The resulting equation will be $x = f^{-1}(y)$, which provides a formula for f^{-1} with y as the independent variable.

Step 4. If y is acceptable as the independent variable for the inverse function, then you are done, but if you want to have x as the independent variable, then you need to interchange x and y in the equation $x = f^{-1}(y)$ to obtain $y = f^{-1}(x)$.

An alternative way to obtain a formula for $f^{-1}(x)$ with x as the independent variable is to reverse the roles of x and y at the outset and solve the equation $x = f(y)$ for y as a function of x.

▶ **Example 4** Find a formula for the inverse of $f(x) = \sqrt{3x-2}$ with x as the independent variable, and state the domain of f^{-1}.

Solution. Following the procedure stated above, we first write

$$y = \sqrt{3x-2}$$

Then we solve this equation for x as a function of y:

$$y^2 = 3x - 2$$
$$x = \tfrac{1}{3}(y^2 + 2)$$

Since we want x to be the independent variable, we reverse x and y in the last equation to produce the formula

$$f^{-1}(x) = \tfrac{1}{3}(x^2 + 2) \tag{5}$$

We know from (4) that the domain of f^{-1} is the range of f. In general, this need not be the same as the natural domain of the fomula for f^{-1}. Indeed, in this example the natural domain of (5) is $(-\infty, +\infty)$, whereas the range of $f(x) = \sqrt{3x-2}$ is $[0, +\infty)$. Thus, if we want to make the domain of f^{-1} clear, we must express it explicitly by rewriting (5) as

$$f^{-1}(x) = \tfrac{1}{3}(x^2 + 2), \quad x \geq 0 \blacktriangleleft$$

EXISTENCE OF INVERSE FUNCTIONS

The procedure we gave above for finding the inverse of a function f was based on solving the equation $y = f(x)$ for x as a function of y. This procedure can fail for two reasons—the function f may not have an inverse, or it may have an inverse but the equation $y = f(x)$ cannot be solved explicitly for x as a function of y. Thus, it is important to establish conditions that ensure the existence of an inverse, even if it cannot be found explicitly.

If a function f has an inverse, then it must assign distinct outputs to distinct inputs. For example, the function $f(x) = x^2$ cannot have an inverse because it assigns the same value to $x = 2$ and $x = -2$, namely, $f(2) = f(-2) = 4$. Thus, if $f(x) = x^2$ were to have an inverse, then the equation $f(2) = 4$ would imply that $f^{-1}(4) = 2$, and the equation $f(-2) = 4$ would imply that $f^{-1}(4) = -2$. But this is impossible because $f^{-1}(4)$ cannot have two different values. Another way to see that $f(x) = x^2$ has no inverse is to attempt to find the inverse by solving the equation $y = x^2$ for x as a function of y. We run into trouble immediately because the resulting equation $x = \pm\sqrt{y}$ does not express x as a *single* function of y.

A function that assigns distinct outputs to distinct inputs is said to be ***one-to-one*** or ***invertible***, so we know from the preceding discussion that if a function f ~~has an~~ inverse, then it must be one-to-one. The converse is also true, thereby establishing the following theorem.

1.5.3 THEOREM. *A function has an inverse if and only if it is one-to-one.*

Stated algebraically, a function f is one-to-one if and only if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$; stated geometrically, a function f is one-to-one if and only if the graph of $y = f(x)$ is cut at most once by any horizontal line (Figure 1.5.3). The latter statement together with Theorem 1.5.3 provides the following geometric test for determining whether a function has an inverse.

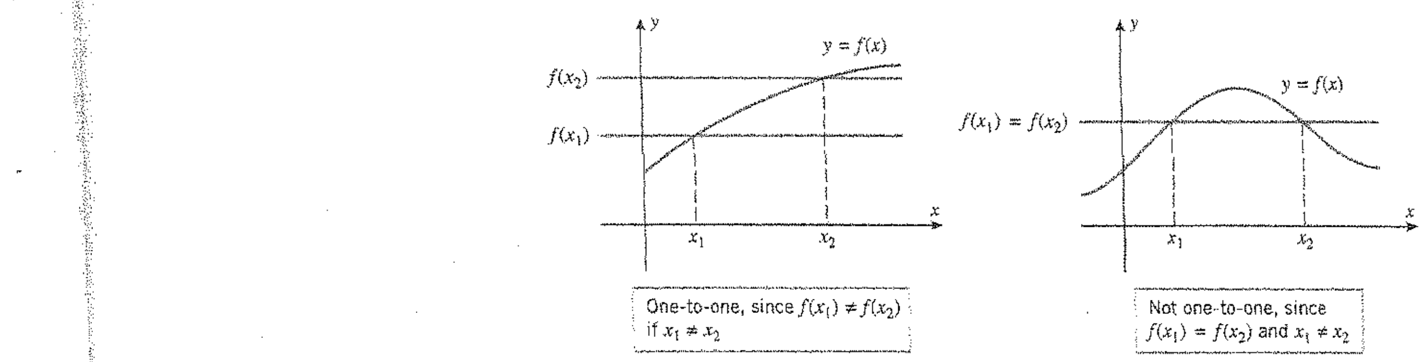


Figure 1.5.3

1.5.4 THEOREM (*The Horizontal Line Test*). *A function has an inverse function if and only if its graph is cut at most once by any horizontal line.*

▶ **Example 5** Use the horizontal line test to show that $f(x) = x^2$ has no inverse but that $f(x) = x^3$ does.

Solution. Figure 1.5.4 shows a horizontal line that cuts the graph of $y = x^2$ more than once, so $f(x) = x^2$ is not invertible. Figure 1.5.5 shows that the graph of $y = x^3$ is cut at most once by any horizontal line, so $f(x) = x^3$ is invertible. [Recall from Example 2 that the inverse of $f(x) = x^3$ is $f^{-1}(x) = x^{1/3}$.] ◀

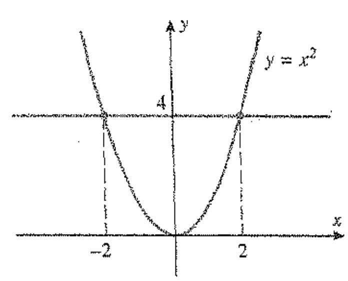


Figure 1.5.4

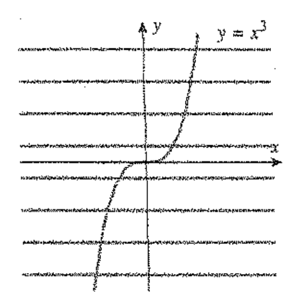


Figure 1.5.5

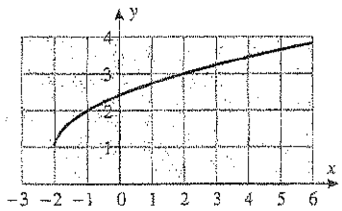


Figure 1.5.6

▶ **Example 6** Explain why the function f that is graphed in Figure 1.5.6 has an inverse, and find $f^{-1}(3)$.

Solution. The function f has an inverse since its graph passes the horizontal line test. To evaluate $f^{-1}(3)$, we view $f^{-1}(3)$ as that number x for which $f(x) = 3$. From the graph we see that $f(2) = 3$, so $f^{-1}(3) = 2$. ◀

The function $f(x) = x^3$ in Figure 1.5.5 is an example of an increasing function. Give an example of a decreasing function and compute its inverse.

Inverse functions are symmetric about y=x

INCREASING OR DECREASING FUNCTIONS ARE INVERTIBLE

A function whose graph is always rising as it is traversed from left to right is said to be an ***increasing function***, and a function whose graph is always falling as it is traversed from left to right is said to be a ***decreasing function***. If x_1 and x_2 are points in the domain of a function f, then f is increasing if

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2$$

and f is decreasing if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2$$

(Figure 1.5.7). It is evident geometrically that increasing and decreasing functions pass the horizontal line test and hence are invertible.

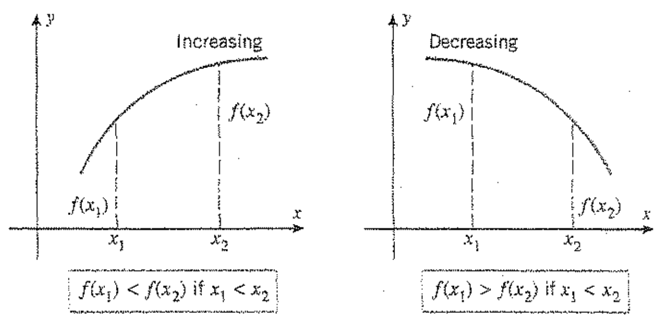
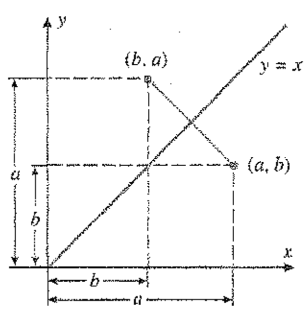


Figure 1.5.7



The points (a, b) and (b, a) are reflections about $y = x$.

Figure 1.5.8

GRAPHS OF INVERSE FUNCTIONS

Our next objective is to explore the relationship between the graphs of f and f^{-1}. For this purpose, it will be desirable to use x as the independent variable for both functions so we can compare the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

If (a, b) is a point on the graph $y = f(x)$, then $b = f(a)$. This is equivalent to the statement that $a = f^{-1}(b)$, which means that (b, a) is a point on the graph of $y = f^{-1}(x)$. In short, reversing the coordinates of a point on the graph of f produces a point on the graph of f^{-1}. Similarly, reversing the coordinates of a point on the graph of f^{-1} produces a point on the graph of f (verify). However, the geometric effect of reversing the coordinates of a point is to reflect that point about the line $y = x$ (Figure 1.5.8), and hence the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of one another about this line (Figure 1.5.9). In summary, we have the following result.

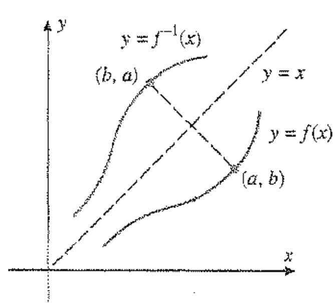


Figure 1.5.9

1.5.5 THEOREM. *If f has an inverse, then the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of one another about the line $y = x$; that is, each graph is the mirror image of the other with respect to that line.*

▶ **Example 7** Figure 1.5.10 shows the graphs of the inverse functions discussed in Examples 2 and 4. ◀

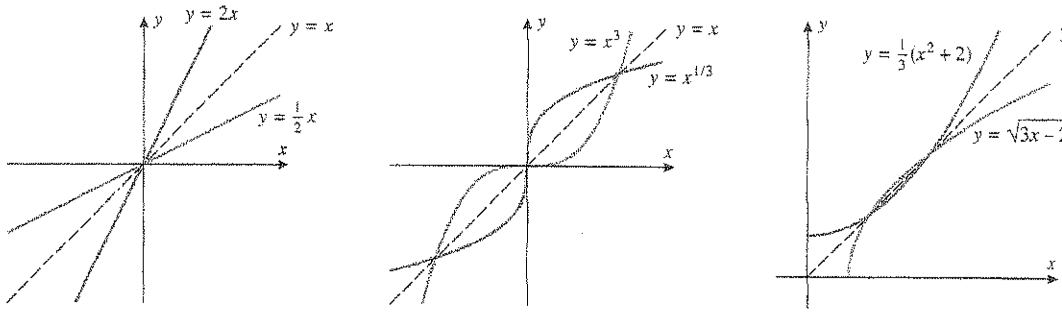


Figure 1.5.10

RESTRICTING DOMAINS FOR INVERTIBILITY

If a function g is obtained from a function f by placing restrictions on the domain of f, then g is called a ***restriction of*** f. Thus, for example, the function

$$g(x) = x^3, \quad x \geq 0$$

is a restriction of the function $f(x) = x^3$. More precisely, it is called the restriction of x^3 to the interval $[0, +\infty)$.

Sometimes it is possible to create an invertible function from a function that is not invertible by restricting the domain appropriately. For example, we showed earlier that $f(x) = x^2$ is not invertible. However, consider the restricted functions

$$f_1(x) = x^2, \quad x \geq 0 \quad \text{and} \quad f_2(x) = x^2, \quad x \leq 0$$

the union of whose graphs is the complete graph of $f(x) = x^2$ (Figure 1.5.11). These restricted functions are each one-to-one (hence invertible), since their graphs pass the horizontal line test. As illustrated in Figure 1.5.12, their inverses are

$$f_1^{-1}(x) = \sqrt{x} \quad \text{and} \quad f_2^{-1}(x) = -\sqrt{x}$$

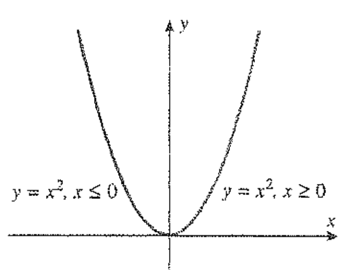


Figure 1.5.11

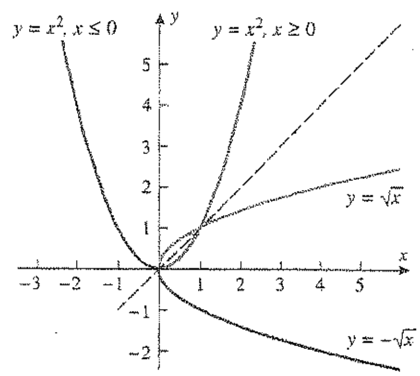


Figure 1.5.12

INVERSE TRIGONOMETRIC FUNCTIONS

A common problem in trigonometry is to find an angle x using a known value of $\sin x$, $\cos x$, or some other trigonometric function. Recall that problems of this type involve the computation of "arc functions" such as $\arcsin x$, $\arccos x$, and so forth. We will conclude this section by studying these arc functions from the viewpoint of general inverse functions.

The six basic trigonometric functions do not have inverses because their graphs repeat periodically and hence do not pass the horizontal line test. To circumvent this problem we will restrict the domains of the trigonometric functions to produce one-to-one functions and then define the "inverse trigonometric functions" to be the inverses of these restricted functions. The top part of Figure 1.5.13 shows geometrically how these restrictions are made for $\sin x$, $\cos x$, $\tan x$, and $\sec x$, and the bottom part of the figure shows the graphs of the corresponding inverse functions

If you have trouble visualizing the correspondence between the top and bottom parts of Figure 1.5.13, keep in mind that a reflection about $y = x$ converts vertical lines into horizontal lines, and vice versa, and converts x-intercepts into y-intercepts, and vice versa.

$$\sin^{-1} x, \quad \cos^{-1} x, \quad \tan^{-1} x, \quad \sec^{-1} x$$

(also denoted by $\arcsin x$, $\arccos x$, $\arctan x$, and $\operatorname{arcsec} x$). Inverses of $\cot x$ and $\csc x$ are of lesser importance and will be considered in the exercises.

LOOK FAMILIAR? TRIG!

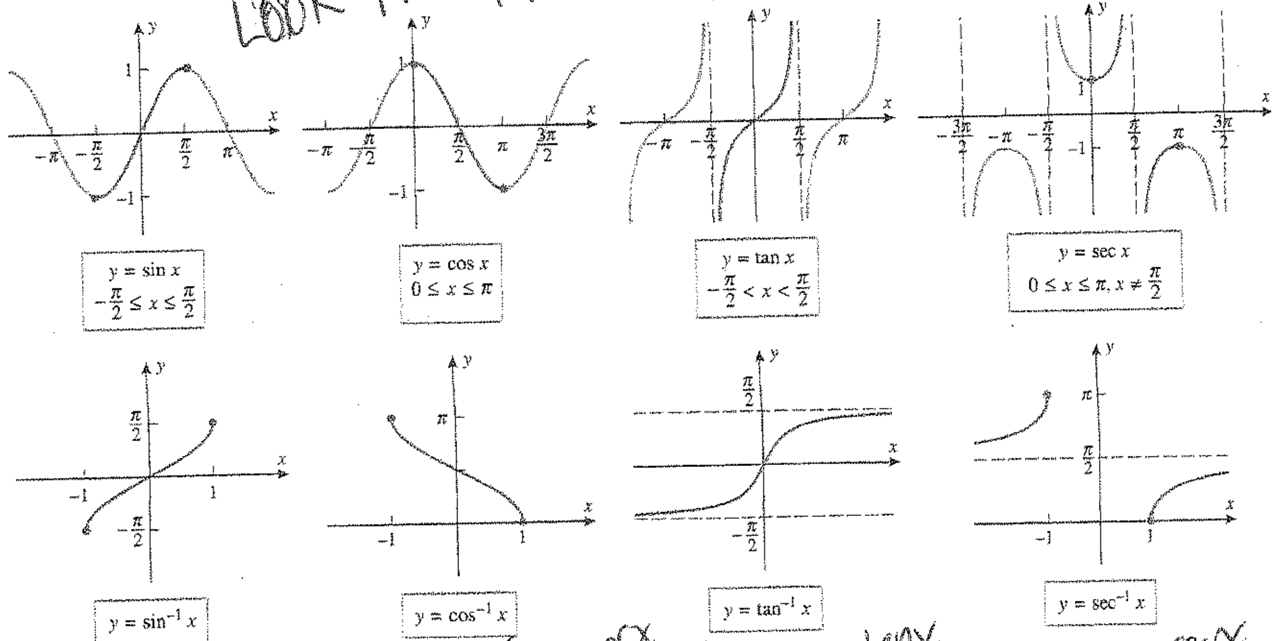


Figure 1.5.13

or y=arcsinx

or y=arccosx

y=arctanx

y=arcsecx

The following formal definitions summarize the preceding discussion.

1.5.6 DEFINITION. The *inverse sine function*, denoted by $\sin^{-1}$, is defined to be the inverse of the restricted sine function

$$\sin x, \quad -\pi/2 \le x \le \pi/2$$

1.5.7 DEFINITION. The *inverse cosine function*, denoted by $\cos^{-1}$, is defined to be the inverse of the restricted cosine function

$$\cos x, \quad 0 \le x \le \pi$$

1.5.8 DEFINITION. The ***inverse tangent function***, denoted by $\tan^{-1}$, is defined to be the inverse of the restricted tangent function

$$\tan x, \quad -\pi/2 < x < \pi/2$$

The notations $\sin^{-1} x, \cos^{-1} x, \ldots$ are reserved exclusively for the inverse trigonometric functions and are not used for reciprocals of the trigonometric functions. If we wanted to express the reciprocal $1/\sin x$ using an exponent, we would write $(\sin x)^{-1}$ and *never* $\sin^{-1} x$.

1.5.9 DEFINITION.* The ***inverse secant function***, denoted by $\sec^{-1}$, is defined to be the inverse of the restricted secant function

$$\sec x, \quad 0 \le x \le \pi \text{ with } x \ne \pi/2$$

Table 1.5.1 summarizes the basic properties of the inverse trigonometric functions we have considered. You should confirm that the domains and ranges listed in this table are consistent with the graphs shown in Figure 1.5.13.

Table 1.5.1

FUNCTION	DOMAIN	RANGE	BASIC RELATIONSHIPS
$\sin^{-1}$	$[-1, 1]$	$[-\pi/2, \pi/2]$	$\sin^{-1}(\sin x) = x$ if $-\pi/2 \le x \le \pi/2$ $\sin(\sin^{-1} x) = x$ if $-1 \le x \le 1$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$	$\cos^{-1}(\cos x) = x$ if $0 \le x \le \pi$ $\cos(\cos^{-1} x) = x$ if $-1 \le x \le 1$
$\tan^{-1}$	$(-\infty, +\infty)$	$(-\pi/2, \pi/2)$	$\tan^{-1}(\tan x) = x$ if $-\pi/2 < x < \pi/2$ $\tan(\tan^{-1} x) = x$ if $-\infty < x < +\infty$
$\sec^{-1}$	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$	$\sec^{-1}(\sec x) = x$ if $0 \le x \le \pi, x \ne \pi/2$ $\sec(\sec^{-1} x) = x$ if $\lvert x\rvert \ge 1$

EVALUATING INVERSE TRIGONOMETRIC FUNCTIONS

A common problem in trigonometry is to find an angle whose sine is known. For example, you might want to find an angle x in radian measure such that

$$\sin x = \tfrac{1}{2} \tag{6}$$

and, more generally, for a given value of y in the interval $-1 \le y \le 1$ you might want to solve the equation

$$\sin x = y \tag{7}$$

Because $\sin x$ repeats periodically, this equation has infinitely many solutions for x; however, if we solve this equation as

$$x = \sin^{-1} y$$

then we isolate the specific solution that lies in the interval $[-\pi/2, \pi/2]$, since this is the range of the inverse sine. For example, Figure 1.5.14 shows four solutions of Equation (6), namely, $-11\pi/6$, $-7\pi/6$, $\pi/6$, and $5\pi/6$. Of these, $\pi/6$ is the solution in the interval $[-\pi/2, \pi/2]$, so

$$\sin^{-1}\left(\tfrac{1}{2}\right) = \pi/6 \tag{8}$$

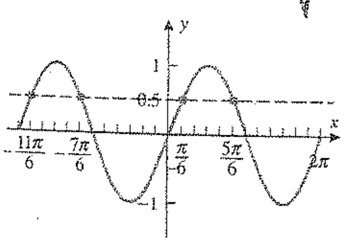


Figure 1.5.14

TECHNOLOGY MASTERY

Refer to the documentation for your calculating utility to determine how to calculate inverse sines, inverse cosines, and inverse tangents; and then confirm Equation (8) numerically by showing that

$$\sin^{-1}(0.5) \approx 0.523598775598\ldots \approx \pi/6$$

*There is no universal agreement on the definition of $\sec^{-1} x$, and some mathematicians prefer to restrict the domain of $\sec x$ so that $0 \le x < \pi/2$ or $\pi \le x < 3\pi/2$, which was the definition used in some earlier editions of this text. Each definition has advantages and disadvantages, but we will use the current definition to conform with the conventions used by the CAS programs *Mathematica*, *Maple*, and *Derive*.

✔ QUICK CHECK EXERCISES 1.5 *(See page 65 for answers.)*

1. In each part, determine whether the function f is one-to-one.
   (a) $f(t)$ is the number of people in line at a movie theater at time t.
   (b) $f(x)$ is the measured high temperature (rounded to the nearest °F) in a city on the xth day of the year.
   (c) $f(v)$ is the weight of v cubic inches of lead.

2. A student enters a number on a calculator, doubles it, adds 8 to the result, divides the sum by 2, subtracts 3 from the quotient, and then cubes the difference. If the resulting number is x, then ________ was the student's original number.

3. If $(3, -2)$ is a point on the graph of an odd invertible function f, then ________ and ________ are points on the graph of f^{-1}.

4. In each part, determine the exact value without using a calculating utility.
   (a) $\sin^{-1}(-1) =$ ________
   (b) $\tan^{-1}(1) =$ ________
   (c) $\sin^{-1}\left(\frac{1}{2}\sqrt{3}\right) =$ ________
   (d) $\cos^{-1}\left(\frac{1}{2}\right) =$ ________
   (e) $\sec^{-1}(-2) =$ ________

5. In each part, determine the exact value without using a calculating utility.
   (a) $\sin^{-1}(\sin \pi/7) =$ ________
   (b) $\sin^{-1}(\sin 5\pi/7) =$ ________
   (c) $\tan^{-1}(\tan 13\pi/6) =$ ________
   (d) $\cos^{-1}(\cos 12\pi/7) =$ ________

EXERCISE SET 1.5 Graphing Utility

1. In (a)–(d), determine whether f and g are inverse functions.
   (a) $f(x) = 4x,\ g(x) = \frac{1}{4}x$
   (b) $f(x) = 3x + 1,\ g(x) = 3x - 1$
   (c) $f(x) = \sqrt[3]{x-2},\ g(x) = x^3 + 2$
   (d) $f(x) = x^4,\ g(x) = \sqrt[4]{x}$

2. Check your answers to Exercise 1 with a graphing utility by determining whether the graphs of f and g are reflections of one another about the line $y = x$.

3. In each part, use the horizontal line test to determine whether the function f is one-to-one.
   (a) $f(x) = 3x + 2$ (b) $f(x) = \sqrt{x-1}$
   (c) $f(x) = |x|$ (d) $f(x) = x^3$
   (e) $f(x) = x^2 - 2x + 2$ (f) $f(x) = \sin x$

4. In each part, generate the graph of the function f with a graphing utility, and determine whether f is one-to-one.
   (a) $f(x) = x^3 - 3x + 2$ (b) $f(x) = x^3 - 3x^2 + 3x - 1$

FOCUS ON CONCEPTS

5. In each part, determine whether the function f defined by the table is one-to-one.

(a)

x	1	2	3	4	5	6
$f(x)$	−2	−1	0	1	2	3

(b)

x	1	2	3	4	5	6
$f(x)$	4	−7	6	−3	1	4

6. A face of a broken clock lies in the xy-plane with the center of the clock at the origin and 3:00 in the direction of the positive x-axis. When the clock broke, the tip of the hour hand stopped on the graph of $y = f(x)$, where f is a function that satisfies $f(0) = 0$.
   (a) Are there any times of the day that cannot appear in such a configuration? Explain.
   (b) How does your answer to part (a) change if f must be an invertible function?
   (c) How do your answers to parts (a) and (b) change if it was the tip of the minute hand that stopped on the graph of f?

7. (a) The accompanying figure shows the graph of a function f over its domain $-8 \le x \le 8$. Explain why f has an inverse, and use the graph to find $f^{-1}(2)$, $f^{-1}(-1)$, and $f^{-1}(0)$.
   (b) Find the domain and range of f^{-1}.
   (c) Sketch the graph of f^{-1}.

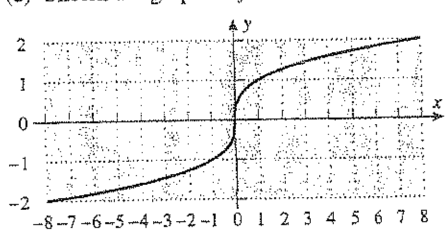


Figure Ex-7

8. (a) Explain why the function f graphed in the accompanying figure has no inverse function on its domain $-3 \le x \le 4$.
   (b) Subdivide the domain into three adjacent intervals on each of which the function f has an inverse.

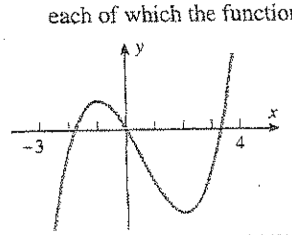


Figure Ex-8

Show #'s in own inverse

9–17 Find a formula for $f^{-1}(x)$.

9. $f(x) = 7x - 6$
10. $f(x) = \dfrac{x+1}{x-1}$
11. $f(x) = 3x^3 - 5$
12. $f(x) = \sqrt[5]{4x+2}$
13. $f(x) = \sqrt[3]{2x-1}$
14. $f(x) = 5/(x^2+1), \quad x \geq 0$
15. $f(x) = 3/x^2, \quad x < 0$
16. $f(x) = \begin{cases} 2x, & x \leq 0 \\ x^2, & x > 0 \end{cases}$
17. $f(x) = \begin{cases} 5/2 - x, & x < 2 \\ 1/x, & x \geq 2 \end{cases}$
18. Find a formula for $p^{-1}(x)$, given that

$$p(x) = x^3 - 3x^2 + 3x - 1$$

19–23 Find a formula for $f^{-1}(x)$, ~~and state the domain of the function f^{-1}~~.

19. $f(x) = (x+2)^4, \quad x \geq 0$
20. $f(x) = \sqrt{x+3}$
21. $f(x) = -\sqrt{3-2x}$
22. $f(x) = 3x^2 + 5x - 2, \quad x \geq 0$
23. $f(x) = x - 5x^2, \quad x \geq 1$

FOCUS ON CONCEPTS

24. The formula $F = \frac{9}{5}C + 32$, where $C \geq -273.15$ expresses the Fahrenheit temperature F as a function of the Celsius temperature C.
    (a) Find a formula for the inverse function.
    (b) In words, what does the inverse function tell you?
    (c) Find the domain and range of the inverse function.
25. (a) One meter is about 6.214×10^{-4} miles. Find a formula $y = f(x)$ that expresses a length y in meters as a function of the same length x in miles.
    (b) Find a formula for the inverse of f.
    (c) Describe what the formula $x = f^{-1}(y)$ tells you in practical terms.
26. Let $f(x) = x^2$, $x > 1$, and $g(x) = \sqrt{x}$.
    (a) Show that $f(g(x)) = x$, $x > 1$, and $g(f(x)) = x$, $x > 1$.
    (b) Show that f and g are *not* inverses by showing that the graphs of $y = f(x)$ and $y = g(x)$ are not reflections of one another about $y = x$.
    (c) Do parts (a) and (b) contradict one another? Explain.
27. (a) Show that $f(x) = (3-x)/(1-x)$ is its own inverse.
    (b) What does the result in part (a) tell you about the graph of f?
28. Let $f(x) = ax^2 + bx + c$, $a > 0$. Find f^{-1} if the domain of f is restricted to
    (a) $x \geq -b/(2a)$ (b) $x \leq -b/(2a)$.
29. Let $f(x) = 2x^3 + 5x + 3$. Find x if $f^{-1}(x) = 1$.
30. Let $f(x) = \dfrac{x^3}{x^2+1}$. Find x if $f^{-1}(x) = 2$.
31. Prove that if $a^2 + bc \neq 0$, then the graph of

$$f(x) = \frac{ax+b}{cx-a}$$

is symmetric about the line $y = x$.

32. (a) Prove: If f and g are one-to-one, then so is the composition $f \circ g$.
    (b) Prove: If f and g are one-to-one, then

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$

33. Sketch the graph of a function that is one-to-one on $(-\infty, +\infty)$, yet not increasing on $(-\infty, +\infty)$ and not decreasing on $(-\infty, +\infty)$.
34. Prove: A one-to-one function f cannot have two different inverses.
35. Given that $\theta = \tan^{-1}\left(\frac{4}{3}\right)$, find the exact values of $\sin\theta$, $\cos\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$. TRIG review!!
36. Given that $\theta = \sec^{-1} 2.6$, find the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, and $\csc\theta$.
37. For which values of x is it true that
    (a) $\cos^{-1}(\cos x) = x$ (b) $\cos(\cos^{-1} x) = x$
    (c) $\tan^{-1}(\tan x) = x$ (d) $\tan(\tan^{-1} x) = x$?

38–39 Find the exact value of the given quantity.

38. $\sec\left[\sin^{-1}\left(-\frac{3}{4}\right)\right]$
39. $\sin\left[2\cos^{-1}\left(\frac{3}{5}\right)\right]$

40–41 Complete the identities using the triangle method (Figure 1.5.15).

40. (a) $\sin(\cos^{-1} x) = ?$ (b) $\tan(\cos^{-1} x) = ?$
    (c) $\csc(\tan^{-1} x) = ?$ (d) $\sin(\tan^{-1} x) = ?$
41. (a) $\cos(\tan^{-1} x) = ?$ (b) $\tan(\cos^{-1} x) = ?$
    (c) $\sin(\sec^{-1} x) = ?$ (d) $\cot(\sec^{-1} x) = ?$
42. (a) Use a calculating utility set to radian measure to make tables of values of $y = \sin^{-1} x$ and $y = \cos^{-1} x$ for $x = -1, -0.8, -0.6, \ldots, 0, 0.2, \ldots, 1$. Round your answers to two decimal places.
    (b) Plot the points obtained in part (a), and use the points to sketch the graphs of $y = \sin^{-1} x$ and $y = \cos^{-1} x$. Confirm that your sketches agree with those in Figure 1.5.13.
    (c) Use your graphing utility to graph $y = \sin^{-1} x$ and $y = \cos^{-1} x$; confirm that the graphs agree with those in Figure 1.5.13.
43. In each part, sketch the graph and check your work with a graphing utility.
    (a) $y = \sin^{-1} 2x$ (b) $y = \tan^{-1} \frac{1}{2}x$

(a) the maximum number of daylight hours at Fairbanks to one decimal place

(b) the minimum number of daylight hours at Fairbanks to one decimal place.

Source: This problem was adapted from *TEAM, A Path to Applied Mathematics*, The Mathematical Association of America, Washington, D.C., 1985.

TRIG Review!

54. The *law of cosines* states that

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

where a, b, and c are the lengths of the sides of a triangle and θ is the angle formed by sides a and b. Find θ, to the nearest degree, for the triangle with $a = 2$, $b = 3$, and $c = 4$.

55. An airplane is flying at a constant height of 3000 ft above water at a speed of 400 ft/s. The pilot is to release a survival package so that it lands in the water at a sighted point P. If air resistance is neglected, then the package will follow a parabolic trajectory whose equation relative to the coordinate system in the accompanying figure is

$$y = 3000 - \frac{g}{2v^2}x^2$$

where g is the acceleration due to gravity and v is the speed of the airplane. Using $g = 32\text{ ft/s}^2$, find the "line of sight" angle θ, to the nearest degree, that will result in the package hitting the target point.

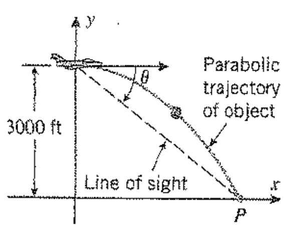


Figure Ex-55

56. A camera is positioned x feet from the base of a missile launching pad (see the accompanying figure). If a missile of length a feet is launched vertically, show that when the base of the missile is b feet above the camera lens, the angle θ subtended at the lens by the missile is

$$\theta = \cot^{-1}\frac{x}{a+b} - \cot^{-1}\frac{x}{b}$$

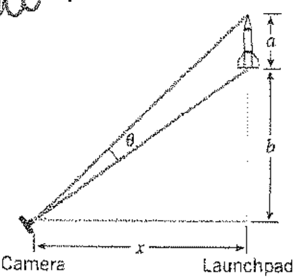


Figure Ex-56

57. Prove:
(a) $\sin^{-1}(-x) = -\sin^{-1}x$
(b) $\tan^{-1}(-x) = -\tan^{-1}x$.

58. Prove:
(a) $\cos^{-1}(-x) = \pi - \cos^{-1}x$
(b) $\sec^{-1}(-x) = \pi - \sec^{-1}x$.

59. Prove:
(a) $\sin^{-1}x = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}} \quad (|x| < 1)$
(b) $\cos^{-1}x = \dfrac{\pi}{2} - \tan^{-1}\dfrac{x}{\sqrt{1-x^2}} \quad (|x| < 1)$.

60. Prove:

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

provided $-\pi/2 < \tan^{-1}x + \tan^{-1}y < \pi/2$. [*Hint:* Use an identity for $\tan(\alpha + \beta)$.]

61. Use the result in Exercise 60 to show that
(a) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \pi/4$
(b) $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \pi/4$.

62. Use identities (9) and (12) to obtain identity (16).

✔ QUICK CHECK ANSWERS 1.5

1. (a) not one-to-one (b) not one-to-one (c) one-to-one 2. $\sqrt[3]{x} - 1$ 3. $(-2, 3)$; $(2, -3)$ 4. (a) $-\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/3$ (e) $2\pi/3$ 5. (a) $\pi/7$ (b) $2\pi/7$ (c) $\pi/6$ (d) $2\pi/7$

1.6 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

When logarithms were introduced in the seventeenth century as a computational tool, they provided scientists of that period computing power that was previously unimaginable. Although computers and calculators have replaced logarithm tables for numerical calculations, the logarithmic functions have wide-ranging applications in mathematics and science. In this section we will review some properties of exponents and logarithms and then use our work on inverse functions to develop results about exponential and logarithmic functions.

$y = b^x$ about the y-axis. This is because replacing x by $-x$ in the equation $y = b^x$ yields

$$y = b^{-x} = (1/b)^x$$

The figure also conveys that for $b > 1$, the larger the base, the more rapidly the function $f(x) = b^x$ increases for $x > 0$.

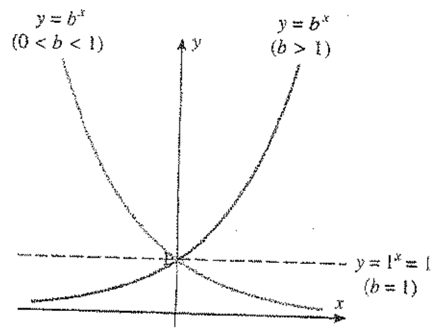


Figure 1.6.1

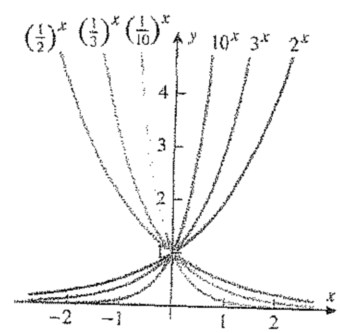


Figure 1.6.2 The family $y = b^x (b > 0)$

If $b > 0$, then $f(x) = b^x$ is defined and has a real value for every real value of x, so the natural domain of every exponential function is $(-\infty, +\infty)$. If $b > 0$ and $b \neq 1$, then as noted earlier the graph of $y = b^x$ increases indefinitely as it is traversed in one direction and decreases toward zero but never reaches zero as it is traversed in the other direction. This implies that the range of $f(x) = b^x$ is $(0, +\infty)$.*

▶ **Example 1** Sketch the graph of the function $f(x) = 1 - 2^x$ and find its domain and range.

Solution. Start with a graph of $y = 2^x$. Reflect this graph across the x-axis to obtain the graph of $y = -2^x$, then translate that graph upward by 1 unit to obtain the graph of $y = 1 - 2^x$ (Figure 1.6.3). The dashed line in the third part of Figure 1.6.3 is a horizontal asymptote for the graph. You should be able to see from the graph that the domain of f is $(-\infty, +\infty)$ and the range is $(-\infty, 1)$. ◀

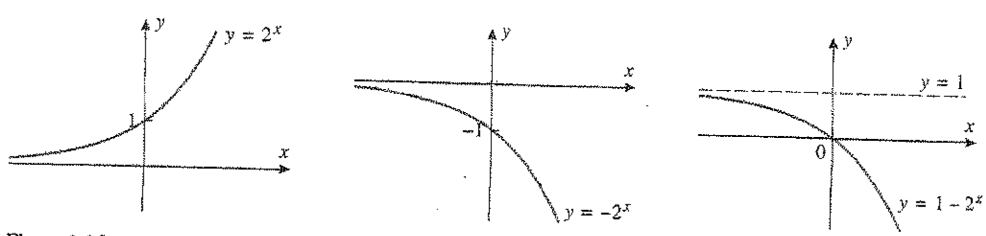


Figure 1.6.3

THE NATURAL EXPONENTIAL FUNCTION

The use of the letter e is in honor of the Swiss mathematician Leonhard Euler (biography on p. 3) who is credited with recognizing the mathematical importance of this constant.

Among all possible bases for exponential functions there is one particular base that plays a special role in calculus. That base, denoted by the letter e, is a certain irrational number whose value to six decimal places is

$$e \approx 2.718282 \qquad (2)$$

*We are assuming without proof that the graph of $y = b^x$ is a curve without breaks, gaps, or holes.

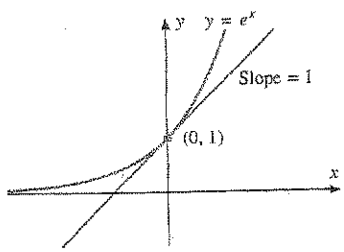


Figure 1.6.4 The tangent line to the graph of $y = e^x$ at (0, 1) has slope 1.

This base is important in calculus because, as we will prove later, $b = e$ is the only base for which the slope of the tangent line* to the curve $y = b^x$ at any point P on the curve is equal to the y-coordinate at P. Thus, for example, the tangent line to $y = e^x$ at (0, 1) has slope 1 (Figure 1.6.4).

The function $f(x) = e^x$ is called the ***natural exponential function***. Since the number e is between 2 and 3, the graph of $y = e^x$ fits between the graphs of $y = 2^x$ and $y = 3^x$ as shown in Figure 1.6.5. To simplify typography, the natural exponential function is sometimes written as $\exp(x)$ in which case the relationship $e^{x_1+x_2} = e^{x_1}e^{x_2}$ would be expressed as

$$\exp(x_1 + x_2) = \exp(x_1)\exp(x_2)$$

TECHNOLOGY MASTERY

Your technology utility should have keys or commands for approximating e and for graphing the natural exponential function. Read your documentation on how to do this and use your utility to confirm (2) and to generate the graphs in Figure 1.6.5.

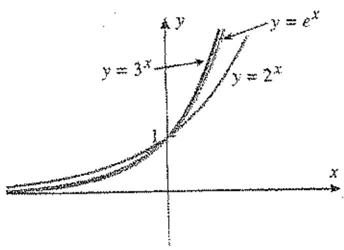


Figure 1.6.5

The constant e also arises in the context of the graph of the equation

$$y = \left(1 + \frac{1}{x}\right)^x \tag{3}$$

As shown in Figure 1.6.6, $y = e$ is a horizontal asymptote of this graph. As a result, the value of e can be approximated to any degree of accuracy by evaluating (3) for x sufficiently large in absolute value (Table 1.6.2).

Recall from Alg - horizontal asymptote.

horizontal asymptote

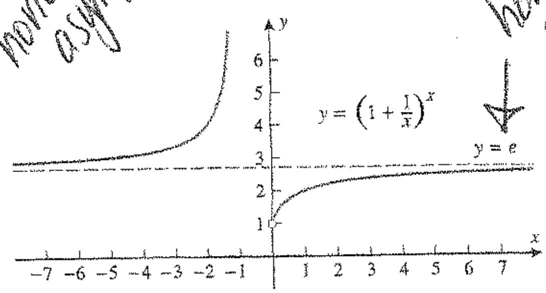


Figure 1.6.6

Table 1.6.2
APPROXIMATIONS OF e BY $(1 + 1/x)^x$ FOR INCREASING VALUES OF x

x	$1 + \frac{1}{x}$	$\left(1 + \frac{1}{x}\right)^x$
1	2	≈ 2.000000
10	1.1	2.593742
100	1.01	2.704814
1000	1.001	2.716924
10,000	1.0001	2.718146
100,000	1.00001	2.718268
1,000,000	1.000001	2.718280

LOGARITHMIC FUNCTIONS

Recall from algebra that a logarithm is an exponent. More precisely, if $b > 0$ and $b \neq 1$, then for a positive value of x the expression

$$\log_b x$$

(read "the logarithm to the base b of x") denotes that exponent to which b must be raised to produce x. Thus, for example,

$$\log_{10} 100 = 2, \quad \log_{10}(1/1000) = -3, \quad \log_2 16 = 4, \quad \log_b 1 = 0, \quad \log_b b = 1$$

$$\boxed{10^2 = 100} \quad \boxed{10^{-3} = 1/1000} \quad \boxed{2^4 = 16} \quad \boxed{b^0 = 1} \quad \boxed{b^1 = b}$$

Logarithms with base 10 are called ***common logarithms*** and are often written without explicit reference to the base. Thus, the symbol $\log x$ generally denotes $\log_{10} x$.

We call the function $f(x) = \log_b x$ the ***logarithmic function with base b***.

*The precise definition of a tangent line will be discussed later. For now your intuition will suffice.

Logarithmic functions can also be viewed as inverses of exponential functions. To see why this is so, observe from Figure 1.6.1 that if $b > 0$ and $b \neq 1$, then the graph of $f(x) = b^x$ passes the horizontal line test, so b^x has an inverse. We can find a formula for this inverse with x as the independent variable by solving the equation

$$x = b^y$$

for y as a function of x. But this equation states that y is the logarithm to the base b of x, so it can be rewritten as

$$y = \log_b x$$

Thus, we have established the following result.

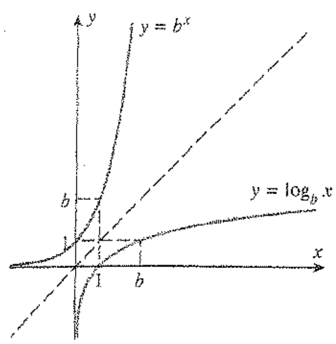


Figure 1.6.7 The functions b^x and $\log_b x$ are inverses.

1.6.1 THEOREM. *If $b > 0$ and $b \neq 1$, then b^x and $\log_b x$ are inverse functions.*

It follows from this theorem that the graphs of $y = b^x$ and $y = \log_b x$ are reflections of one another about the line $y = x$ (see Figure 1.6.7 for the case where $b > 1$). Figure 1.6.8 shows the graphs of $y = \log_b x$ for various values of b. Observe that they all pass through the point $(1, 0)$.

The most important logarithms in applications are those with base e. These are called ***natural logarithms*** because the function $\log_e x$ is the inverse of the natural exponential function e^x. It is standard to denote the natural logarithm of x by $\ln x$ (read "ell en of x"), rather than $\log_e x$. For example,

$$\underbrace{\ln 1 = 0,}_{\text{Since } e^0 = 1} \quad \underbrace{\ln e = 1,}_{\text{Since } e^1 = e} \quad \underbrace{\ln 1/e = -1,}_{\text{Since } e^{-1} = 1/e} \quad \underbrace{\ln(e^2) = 2}_{\text{Since } e^2 = e^2}$$

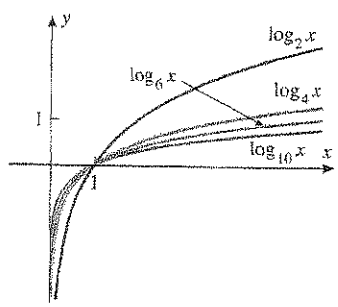


Figure 1.6.8 The family $y = \log_b x$ $(b > 1)$

In general,

$$y = \ln x \quad \text{if and only if} \quad x = e^y$$

As shown in Table 1.6.3, the inverse relationship between b^x and $\log_b x$ produces a correspondence between some basic properties of those functions.

TECHNOLOGY MASTERY

Use your graphing utility to generate the graphs of $y = \ln x$ and $y = \log x$.

Table 1.6.3
CORRESPONDENCE BETWEEN PROPERTIES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

PROPERTY OF b^x	PROPERTY OF $\log_b x$
$b^0 = 1$	$\log_b 1 = 0$
$b^1 = b$	$\log_b b = 1$
Range is $(0, +\infty)$	Domain is $(0, +\infty)$
Domain is $(-\infty, +\infty)$	Range is $(-\infty, +\infty)$

It also follows from the cancellation properties of inverse functions [see (3) in Section 1.5] that

$$\begin{aligned} \log_b(b^x) &= x \quad \text{for all real values of } x \\ b^{\log_b x} &= x \quad \text{for } x > 0 \end{aligned} \tag{4}$$

In the special case where $b = e$, these equations become

$$\begin{aligned} \ln(e^x) &= x \quad \text{for all real values of } x \\ e^{\ln x} &= x \quad \text{for } x > 0 \end{aligned} \tag{5}$$

In words, the functions b^x and $\log_b x$ cancel out the effect of one another when composed in either order; for example,

$$\log 10^x = x, \quad 10^{\log x} = x, \quad \ln e^x = x, \quad e^{\ln x} = x, \quad \ln e^5 = 5, \quad e^{\ln \pi} = \pi$$

SOLVING EQUATIONS INVOLVING EXPONENTIALS AND LOGARITHMS

You should be familiar with the following properties of logarithms from your earlier studies.

1.6.2 THEOREM (*Algebraic Properties of Logarithms*). *If $b > 0$, $b \neq 1$, $a > 0$, $c > 0$, and r is any real number, then:*

(*a*) $\log_b(ac) = \log_b a + \log_b c$ — Product property

(*b*) $\log_b(a/c) = \log_b a - \log_b c$ — Quotient property

(*c*) $\log_b(a^r) = r \log_b a$ — Power property

(*d*) $\log_b(1/c) = -\log_b c$ — Reciprocal property

Expressions of the form $\log_b(u+v)$ and $\log_b(u-v)$ have no useful simplifications. In particular,

$$\log_b(u+v) \neq \log_b(u) + \log_b(v)$$
$$\log_b(u-v) \neq \log_b(u) - \log_b(v)$$

These properties are often used to expand a single logarithm into sums, differences, and multiples of other logarithms and, conversely, to condense sums, differences, and multiples of logarithms into a single logarithm. For example,

$$\log \frac{xy^5}{\sqrt{z}} = \log xy^5 - \log \sqrt{z} = \log x + \log y^5 - \log z^{1/2} = \log x + 5 \log y - \tfrac{1}{2} \log z$$

$$5 \log 2 + \log 3 - \log 8 = \log 32 + \log 3 - \log 8 = \log \frac{32 \cdot 3}{8} = \log 12$$

$$\tfrac{1}{3} \ln x - \ln(x^2 - 1) + 2 \ln(x+3) = \ln x^{1/3} - \ln(x^2 - 1) + \ln(x+3)^2 = \ln \frac{\sqrt[3]{x}(x+3)^2}{x^2 - 1}$$

The inverse relationship between logarithmic and exponential functions provides the following useful result for solving equations involving natural exponentials and logarithms:

$$y = e^x \text{ is equivalent to } x = \ln y \text{ if } y > 0 \text{ and } x \text{ is any real number} \qquad (6)$$

More generally, if $b > 0$ and $b \neq 1$, then

$$y = b^x \text{ is equivalent to } x = \log_b y \text{ if } y > 0 \text{ and } x \text{ is any real number} \qquad (7)$$

An equation of the form $\log_b x = k$ can be solved for x by rewriting it in the exponential form $x = b^k$, and an equation of the form $b^x = k$ can be solved by rewriting it in the logarithm form $x = \log_b k$. Alternatively, the equation $b^x = k$ can be solved by taking *any* logarithm of both sides (but usually log or ln) and applying part (*c*) of Theorem 1.6.2. These ideas are illustrated in the following example.

▶ Example 2 Find x such that

(a) $\log x = \sqrt{2}$ (b) $\ln(x+1) = 5$ (c) $5^x = 7$

Solution (a). Converting the equation to exponential form yields

$$x = 10^{\sqrt{2}} \approx 25.95$$

Solution (b). Converting the equation to exponential form yields

$$x + 1 = e^5 \quad \text{or} \quad x = e^5 - 1 \approx 147.41$$

Solution (c). Taking the natural logarithm of both sides and using the power property of logarithms yields

$$x \ln 5 = \ln 7 \quad \text{or} \quad x = \frac{\ln 7}{\ln 5} \approx 1.21 \blacktriangleleft$$

▶ **Example 3** A satellite that requires 7 watts of power to operate at full capacity is equipped with a radioisotope power supply whose power output P in watts is given by the equation

$$P = 75e^{-t/125}$$

where t is the time in days that the supply is used. How long can the satellite operate at full capacity?

Solution. The power P will fall to 7 watts when

$$7 = 75e^{-t/125}$$

The solution for t is as follows:

$$\begin{aligned} 7/75 &= e^{-t/125} \\ \ln(7/75) &= \ln(e^{-t/125}) \\ \ln(7/75) &= -t/125 \\ t &= -125\ln(7/75) \approx 296.4 \end{aligned}$$

so the satellite can operate at full capacity for about 296 days. ◀

Here is a more complicated example.

▶ **Example 4** Solve $\dfrac{e^x - e^{-x}}{2} = 1$ for x.

Solution. Multiplying both sides of the given equation by 2 yields

$$e^x - e^{-x} = 2$$

or equivalently,

$$e^x - \frac{1}{e^x} = 2$$

Multiplying through by e^x yields

$$e^{2x} - 1 = 2e^x \quad \text{or} \quad e^{2x} - 2e^x - 1 = 0$$

This is really a quadratic equation in disguise, as can be seen by rewriting it in the form

$$(e^x)^2 - 2e^x - 1 = 0$$

and letting $u = e^x$ to obtain

$$u^2 - 2u - 1 = 0$$

Solving for u by the quadratic formula yields

$$u = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

or, since $u = e^x$,

$$e^x = 1 \pm \sqrt{2}$$

But e^x cannot be negative, so we discard the negative value $1 - \sqrt{2}$; thus,

$$\begin{aligned} e^x &= 1 + \sqrt{2} \\ \ln e^x &= \ln(1 + \sqrt{2}) \\ x &= \ln(1 + \sqrt{2}) \approx 0.881 \blacktriangleleft \end{aligned}$$

produced a sound level of 92 dB. What is the ratio of the sound intensity of The Who to the sound intensity of a jackhammer?

Solution. Let I_1 and β_1 ($= 120$ dB) denote the intensity and sound level of The Who, and let I_2 and β_2 ($= 92$ dB) denote the intensity and sound level of the jackhammer. Then

$$\begin{aligned}
I_1/I_2 &= (I_1/I_0)/(I_2/I_0)\\
\log(I_1/I_2) &= \log(I_1/I_0) - \log(I_2/I_0)\\
10\log(I_1/I_2) &= 10\log(I_1/I_0) - 10\log(I_2/I_0)\\
10\log(I_1/I_2) &= \beta_1 - \beta_2 = 120 - 92 = 28\\
\log(I_1/I_2) &= 2.8
\end{aligned}$$

Thus, $I_1/I_2 = 10^{2.8} \approx 631$, which tells us that the sound intensity of The Who was 630 times greater than a jackhammer! ◄

EXPONENTIAL AND LOGARITHMIC GROWTH

The growth patterns of e^x and $\ln x$ illustrated in Table 1.6.5 are worth noting. Both functions increase as x increases, but they increase in dramatically different ways—the value of e^x increases extremely rapidly and that of $\ln x$ increases extremely slowly. For example, at $x = 10$ the value of e^x is over 22,000, but at $x = 1000$ the value of $\ln x$ has not even reached 7.

Table 1.6.5

x	e^x	$\ln x$
1	2.72	0.00
2	7.39	0.69
3	20.09	1.10
4	54.60	1.39
5	148.41	1.61
6	403.43	1.79
7	1096.63	1.95
8	2980.96	2.08
9	8103.08	2.20
10	22026.47	2.30
100	2.69×10^{43}	4.61
1000	1.97×10^{434}	6.91

A function f is said to ***increase without bound*** as x increases if the values of $f(x)$ eventually exceed any specified positive number M (no matter how large) as x increases indefinitely. Table 1.6.5 strongly suggests that $f(x) = e^x$ increases without bound, which is consistent with the fact that the range of this function is $(0, +\infty)$. Indeed, if we choose any positive number M, then we will have $e^x = M$ when $x = \ln M$, and since the values of e^x increase as x increases, we will have

$$e^x > M \quad \text{if} \quad x > \ln M$$

(Figure 1.6.9). It is not clear from Table 1.6.5 whether $\ln x$ increases without bound as x increases because the values grow so slowly, but we know this to be so since the range of this function is $(-\infty, +\infty)$. To see this algebraically, let M be any positive number. We will have $\ln x = M$ when $x = e^M$, and since the values of $\ln x$ increase as x increases, we will have

$$\ln x > M \quad \text{if} \quad x > e^M$$

(Figure 1.6.10).

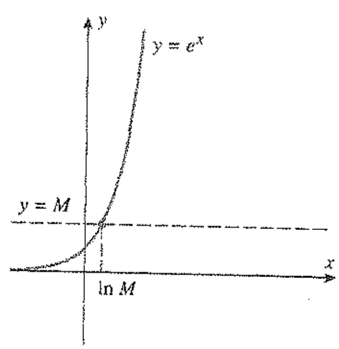


Figure 1.6.9 The value of $y = e^x$ will exceed an arbitrary positive value of M when $x > \ln M$.

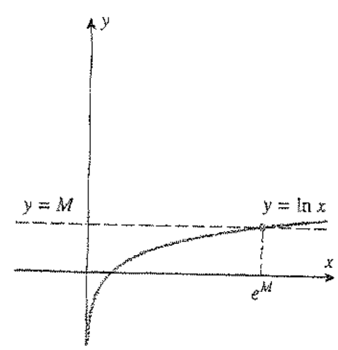


Figure 1.6.10 The value of $y = \ln x$ will exceed an arbitrary positive value of M when $x > e^M$.

$\log\left(\frac{x^2}{x+1}\right) = \log\left(\frac{4}{3}\right)$

✔ QUICK CHECK EXERCISES 1.6 (See page 76 for answers.)

1. The function $y = \left(\frac{1}{2}\right)^x$ has domain ______ and range ______.
2. The function $y = \ln(1 - x)$ has domain ______ and range ______.
3. Express as a power of 4:
(a) 1 (b) 2 (c) $\frac{1}{16}$ (d) $\sqrt{8}$ (e) 5.
4. Solve each equation for x.
(a) $e^x = \frac{1}{2}$ (b) $10^{3x} = 1{,}000{,}000$
(c) $7e^{3x} = 56$
5. Solve each equation for x.
(a) $\ln x = 3$ (b) $\log(x - 1) = 2$
(c) $2\log x - \log(x + 1) = \log 4 - \log 3$

$\log x^2$ $2\log\left(\frac{4}{3}\right)$

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EXERCISE SET 1.6 Graphing Utility

1–2 Simplify the expression without using a calculating utility.

1. (a) $-8^{2/3}$ (b) $(-8)^{2/3}$ (c) $8^{-2/3}$
2. (a) 2^{-4} (b) $4^{1.5}$ (c) $9^{-0.5}$

3–4 Use a calculating utility to approximate the expression. Round your answer to four decimal places.

3. (a) $2^{1.57}$ (b) $5^{-2.1}$
4. (a) $\sqrt[5]{24}$ (b) $\sqrt[8]{0.6}$

5–6 Find the exact value of the expression without using a calculating utility.

5. (a) $\log_2 16$ (b) $\log_2\left(\frac{1}{32}\right)$
(c) $\log_4 4$ (d) $\log_9 3$
6. (a) $\log_{10}(0.001)$ (b) $\log_{10}(10^4)$
(c) $\ln(e^3)$ (d) $\ln(\sqrt{e})$

7–8 Use a calculating utility to approximate the expression. Round your answer to four decimal places.

7. (a) $\log 23.2$ (b) $\ln 0.74$
8. (a) $\log 0.3$ (b) $\ln \pi$

9–10 Use the logarithm properties in Theorem 1.6.2 to rewrite the expression in terms of r, s, and t, where $r = \ln a$, $s = \ln b$, and $t = \ln c$.

9. (a) $\ln a^2\sqrt{bc}$ (b) $\ln \dfrac{b}{a^3 c}$
10. (a) $\ln \dfrac{\sqrt[3]{c}}{ab}$ (b) $\ln \sqrt{\dfrac{ab^3}{c^2}}$

13–15 Rewrite the expression as a single logarithm.

13. $4\log 2 - \log 3 + \log 16$
14. $\frac{1}{2}\log x - 3\log(\sin 2x) + 2$
15. $2\ln(x + 1) + \frac{1}{3}\ln x - \ln(\cos x)$

16–25 Solve for x without using a calculating utility.

16. $\log_{10}(1 + x) = 3$
17. $\log_{10}(\sqrt{x}) = -1$
18. $\ln(x^2) = 4$
19. $\ln(1/x) = -2$
20. $\log_3(3^x) = 7$
21. $\log_5(5^{2x}) = 8$
22. $\log_{10} x^2 + \log_{10} x = 30$
23. $\log_{10} x^{3/2} - \log_{10}\sqrt{x} = 5$
24. $\ln 4x - 3\ln(x^2) = \ln 2$
25. $\ln(1/x) + \ln(2x^3) = \ln 3$

26–31 Solve for x without using a calculating utility. Use the natural logarithm anywhere that logarithms are needed.

26. $3^x = 2$
27. $5^{-2x} = 3$
28. $3e^{-2x} = 5$
29. $2e^{3x} = 7$
30. $e^x - 2xe^x = 0$
31. $xe^{-x} + 2e^{-x} = 0$

32–33 Rewrite the given equation as a quadratic equation in u, where $u = e^x$; then solve for x.

32. $e^{2x} - e^x = 6$
33. $e^{-2x} - 3e^{-x} = -2$

FOCUS ON CONCEPTS

34–36 Sketch the graph of the equation without using a graphing utility.

34. (a) $y = 1 + \ln(x - 2)$ (b) $y = 3 + e^{x-2}$

✔ QUICK CHECK ANSWERS 1.6

1. $(-\infty, +\infty)$; $(0, +\infty)$ 2. $(-\infty, 1)$; $(-\infty, +\infty)$ 3. (a) 4^0 (b) $4^{1/2}$ (c) 4^{-2} (d) $4^{3/4}$ (e) $4^{\log_4 5}$ 4. (a) $\ln\frac{1}{2} = -\ln 2$ (b) 2 (c) $\ln 2$ 5. (a) e^3 (b) 101 (c) 2

1.7 MATHEMATICAL MODELS

38–39 Graph the functions on the same screen of a graphing utility. [Use the change of base formula (8), where needed.]

38. $\ln x,\ e^x,\ \log x,\ 10^x$

39. $\log_2 x,\ \ln x,\ \log_5 x,\ \log x$

40. (a) Derive the general change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

(b) Use the result in part (a) to find the exact value of $(\log_2 81)(\log_3 32)$ without using a calculating utility.

41. Use a graphing utility to estimate the two points of intersection of the graphs of $y = 1.3^x$ and $y = \log_{1.3} x$.

42. The United States public debt D, in billions of dollars, has been modeled as $D = 0.051517(1.1306727)^x$, where x is the number of years since 1900. Based on this model, when did the debt first reach one trillion dollars?

FOCUS ON CONCEPTS

43. (a) Is the curve in the accompanying figure the graph of an exponential function? Explain your reasoning.
(b) Find the equation of an exponential function that passes through the point (4, 2).
(c) Find the equation of an exponential function that passes through the point $(2, \frac{1}{4})$.
(d) Use a graphing utility to generate the graph of an exponential function that passes through the point (2, 5).

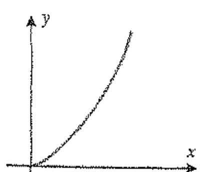


Figure Ex-43

44. (a) Make a conjecture about the general shape of the graph of $y = \log(\log x)$, and sketch the graph of this equation and $y = \log x$ in the same coordinate system.
(b) Check your work in part (a) with a graphing utility.

45. Find the fallacy in the following "proof" that $\frac{1}{8} > \frac{1}{4}$. Multiply both sides of the inequality $3 > 2$ by $\log \frac{1}{2}$ to get

$$3 \log \tfrac{1}{2} > 2 \log \tfrac{1}{2}$$
$$\log \left(\tfrac{1}{2}\right)^3 > \log \left(\tfrac{1}{2}\right)^2$$
$$\log \tfrac{1}{8} > \log \tfrac{1}{4}$$
$$\tfrac{1}{8} > \tfrac{1}{4}$$

46. Prove the four algebraic properties of logarithms in Theorem 1.6.2.

47. If equipment in the satellite of Example 3 requires 15 watts to operate correctly, what is the operational lifetime of the power supply?

48. The equation $Q = 12e^{-0.055t}$ gives the mass Q in grams of radioactive potassium-42 that will remain from some initial quantity after t hours of radioactive decay.
(a) How many grams were there initially?
(b) How many grams remain after 4 hours?
(c) How long will it take to reduce the amount of radioactive potassium-42 to half of the initial amount?

49. The acidity of a substance is measured by its pH value, which is defined by the formula

$$\text{pH} = -\log[H^+]$$

Chemistry

where the symbol $[H^+]$ denotes the concentration of hydrogen ions measured in moles per liter. Distilled water has a pH of 7; a substance is called *acidic* if it has pH < 7 and *basic* if it has pH > 7. Find the pH of each of the following substances and state whether it is acidic or basic.

	SUBSTANCE	$[H^+]$
(a)	Arterial blood	3.9×10^{-8} mol/L
(b)	Tomatoes	6.3×10^{-5} mol/L
(c)	Milk	4.0×10^{-7} mol/L
(d)	Coffee	1.2×10^{-6} mol/L

50. Use the definition of pH in Exercise 49 to find $[H^+]$ in a solution having a pH equal to
(a) 2.44 (b) 8.06.

51. The perceived loudness β of a sound in decibels (dB) is related to its intensity I in watts per square meter (W/m²) by the equation

$$\beta = 10 \log(I/I_0)$$

where $I_0 = 10^{-12}$ W/m². Damage to the average ear occurs at 90 dB or greater. Find the decibel level of each of the following sounds and state whether it will cause ear damage.

	SOUND	I
(a)	Jet aircraft (from 50 ft)	1.0×10^{2} W/m²
(b)	Amplified rock music	1.0 W/m²
(c)	Garbage disposal	1.0×10^{-4} W/m²
(d)	TV (mid volume from 10 ft)	3.2×10^{-5} W/m²

52–54 Use the definition of the decibel level of a sound (see Exercise 51).

52. If one sound is three times as intense as another, how much greater is its decibel level?

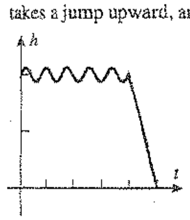
53. According to one source, the noise inside a moving automobile is about 70 dB, whereas an electric blender generates 93 dB. Find the ratio of the intensity of the noise of the blender to that of the automobile.

that's all FOLKS!!!

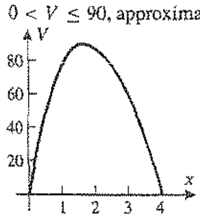
ANSWERS TO ODD-NUMBERED EXERCISES

Exercise Set 1.1 (Page 12)

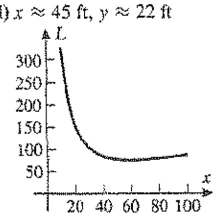
1. (a) $-2.9, -2.0, 2.35, 2.9$ (b) none (c) 0 (d) $-1.75 \le x \le 2.15$ (e) $y_{\max} = 2.8$ at $x = -2.6$; $y_{\min} = -2.2$ at $x = 1.2$
3. (a) yes (b) yes (c) no (d) no
5. (a) 1943 (b) 1960; 4200 (c) no, you need the year's population (d) war, marketing (e) news of health risk, social pressure, anti-smoking campaigns, increased taxation
7. (a) 1999, about \$43,400 (b) 1985, \$37,000 (c) second year
9. (a) -2; 10; 10; 25; 4; $27t^2 - 2$ (b) 0; 4; -4; 6; $2\sqrt{2}$; $f(3t) = 1/3t$ for $t > 1$ and $f(3t) = 6t$ for $t \le 1$
11. (a) $x \ne 3$ (b) $x \le -\sqrt{3}, x \ge \sqrt{3}$ (c) $(-\infty, +\infty)$ (d) $x \ne 0$ (e) $x \ne \left(2n + \frac{1}{2}\right)\pi, n = 0, \pm 1, \pm 2, \ldots$
13. (a) $x \le 3$ (b) $-2 \le x \le 2$ (c) $x \ge 0$ (d) all x (e) all x
15. (a) no; war, pestilence, flood, earthquakes (b) decreases for 8 hours, takes a jump upward, and repeats
17.



19. (a) 2, 4
(b) none
(c) $x \le 2$; $4 \le x$
(d) $y_{\min} = -1$; no maximum
21. $h = L(1 - \cos\theta)$
23. (a) $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 4x + 1, & x \ge 0 \end{cases}$ (b) $g(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \le x \le 1 \\ 2x - 1, & x \ge 1 \end{cases}$
25. (a) $V = (8 - 2x)(15 - 2x)x$
(b) $0 < x < 4$
(c) $0 < V \le 90$, approximately



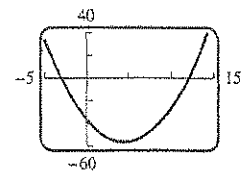
(d) V appears to be maximal for $x \approx 1.7$
27. (a) $L = x + 2y$
(b) $L = x + 2000/x$
(c) $0 < x \le 100$
(d) $x \approx 45$ ft, $y \approx 22$ ft



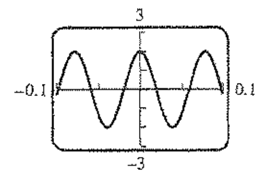
29. (a) $r \approx 3.4, h \approx 13.8$ (b) taller (c) $r \approx 3.1$ cm, $h \approx 16.0$ cm, $C \approx 4.76$ cents
31. (i) $x = 1, -2$ (ii) $g(x) = x + 1$, all x
33. (a) 25°F (b) 13°F (c) 5°F 35. 15°F

Exercise Set 1.2 (Page 24)

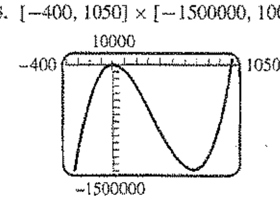
1. (e) 3. (b), (c) 5. $[-3, 3] \times [0, 5]$
9. $[-5, 14] \times [-60, 40]$



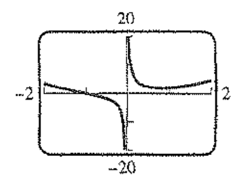
11. $[-0.1, 0.1] \times [-3, 3]$



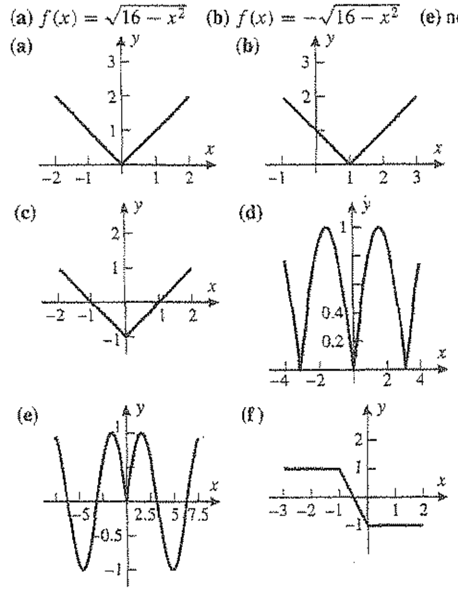
13. $[-400, 1050] \times [-1500000, 10000]$



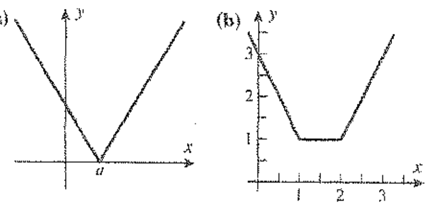
15. $[-2, 2] \times [-20, 20]$

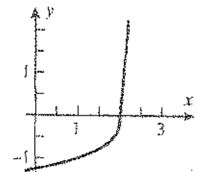


19. (a) $f(x) = \sqrt{16 - x^2}$ (b) $f(x) = -\sqrt{16 - x^2}$ (c) no
21.

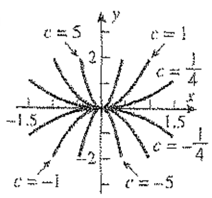


The graph of $y = |f(x)|$ consists of those parts of the graph of $y = f(x)$ which lie above the x-axis, together with the reflections across the x-axis of those parts of the graph of $y = f(x)$ which lie below the x-axis.



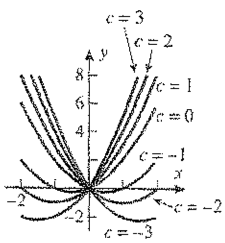


(a)



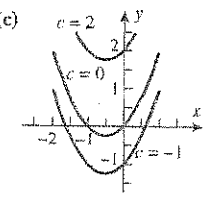
The graph is stretched in the vertical direction, and reflected across the x-axis if $c < 0$.

(b)

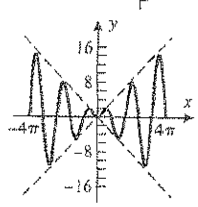


The graph is translated so its vertex is on the parabola $y = -x^2$

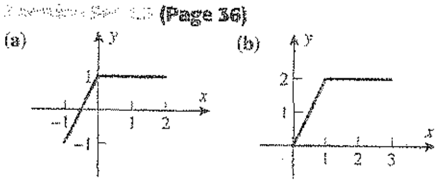
(c)

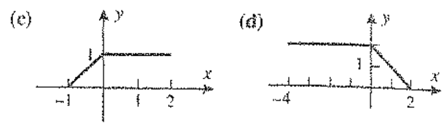


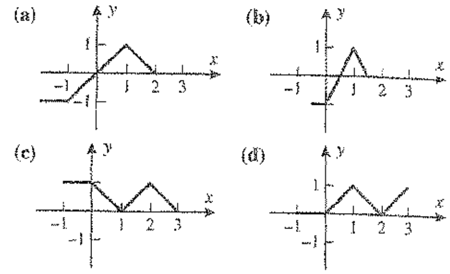
The graph is translated vertically.

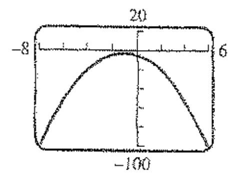


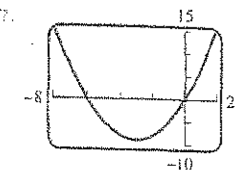
Exercise Set 1.3 (Page 36)

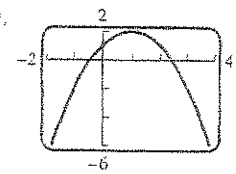


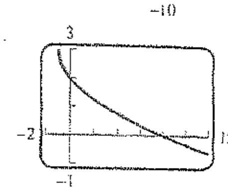


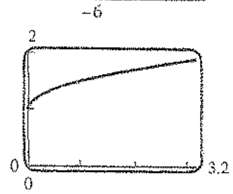


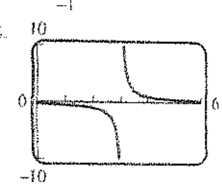


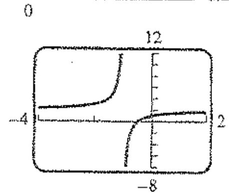


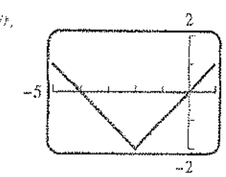


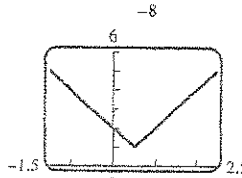


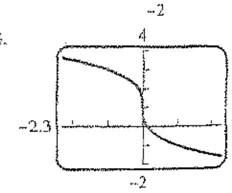


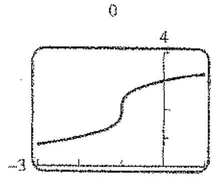




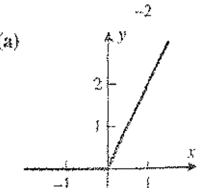








(a)



(b) $y = \begin{cases} 0, & x \le 0 \\ 2x, & x > 0 \end{cases}$

$3\sqrt{x-1}, x \ge 1$; $\sqrt{x-1}, x \ge 1$; $2x - 2, x \ge 1$; $2, x > 1$

(a) 3 (b) 9 (c) 2 (d) 2

33. (a) t^4+1 (b) t^2+4t+5 (c) x^2+4x+5 (d) $\frac{1}{x^2}+1$
(e) $x^2+2xh+h^2+1$ (f) x^2+1 (g) $x+1$ (h) $9x^2+1$

35. $1-x, x \le 1$; $\sqrt{1-x^2}, |x| \le 1$

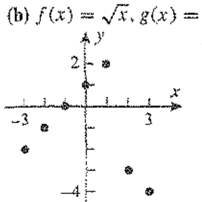
37. $\frac{1}{1-2x}, x \ne \frac{1}{2}, 1$; $-\frac{1}{2x}-\frac{1}{2}, x \ne 0, 1$ 39. $x^{-6}+1$

41. (a) $g(x)=\sqrt{x}, h(x)=x+2$ (b) $g(x)=|x|, h(x)=x^2-3x+5$

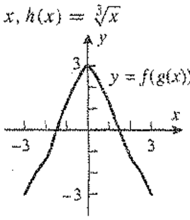
43. (a) $g(x)=x^2, h(x)=\sin x$ (b) $g(x)=3/x, h(x)=5+\cos x$

45. (a) $f(x)=x^3, g(x)=1+\sin x, h(x)=x^2$
(b) $f(x)=\sqrt{x}, g(x)=1-x, h(x)=\sqrt[3]{x}$

47.



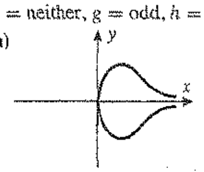
49.



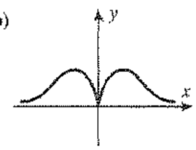
51. $\pm 1.5, \pm 2$ 53. $3w+3x, 6x+3h$ 55. $-\frac{1}{xw}, -\frac{1}{x(x+h)}$

57. f = neither, g = odd, h = even

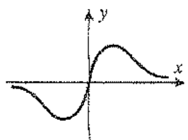
59. (a)



(b)



(c)

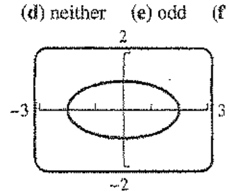


61. (a) even (b) odd (c) odd (d) neither

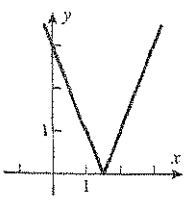
63. (a) even (b) odd (c) even (d) neither (e) odd (f) even

67. (a) y-axis
(b) origin
(c) x-axis, y-axis, origin

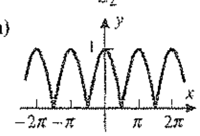
69.



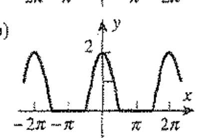
71.



73. (a)



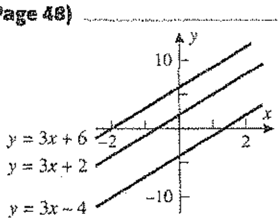
(b)



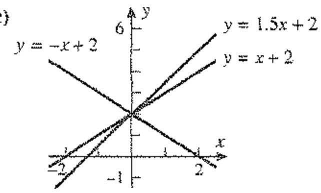
75. yes; $f(x)=x^k, g(x)=x^n$

Exercise Set 1.4 (Page 48)

1. (a) $y=3x+b$ (c)
(b) $y=3x+6$

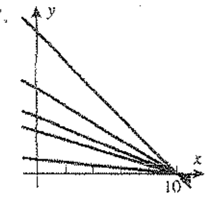


3. (a) $y=mx+2$ (c)
(b) $y=-x+2$



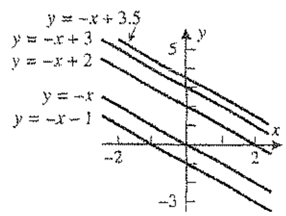
5. $y=\pm\frac{9-x_0x}{\sqrt{9-x_0^2}}$

7.

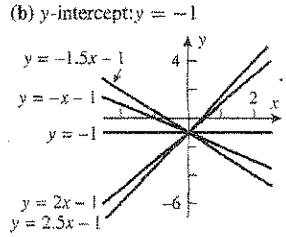


y-intercepts represent current value of item being depreciated.

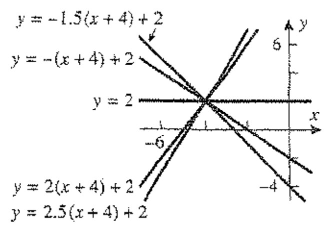
9. (a) slope: -1



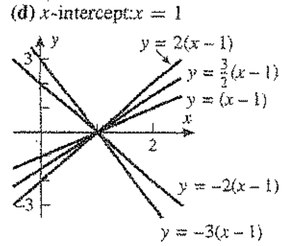
(b) y-intercept: $y=-1$



(c) pass through $(-4, 2)$

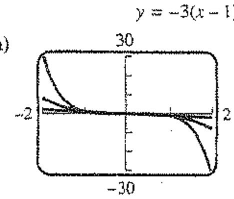


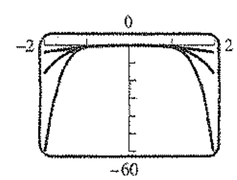
(d) x-intercept: $x=1$



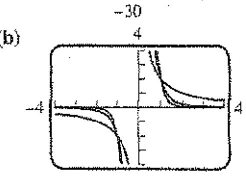
11. (a) VI
(b) IV
(c) III
(d) V
(e) I
(f) II

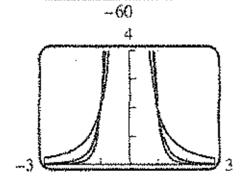
13. (a)



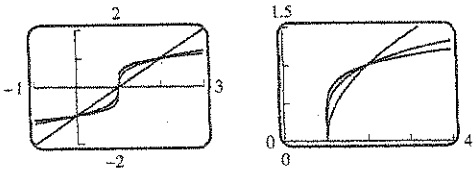


(b)

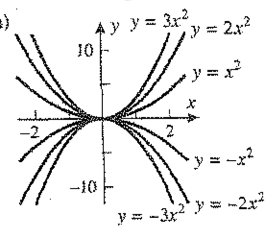




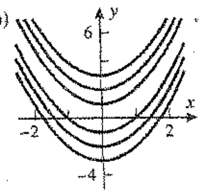
(c)



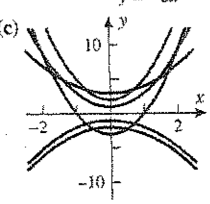
15. (a)



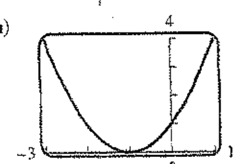
(b)



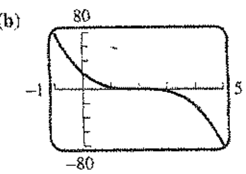
(c)



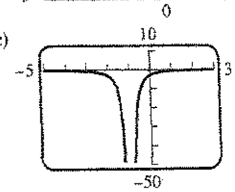
17. (a)



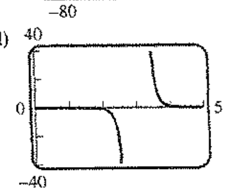
(b)



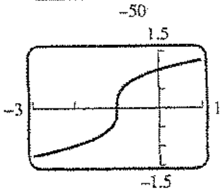
(c)



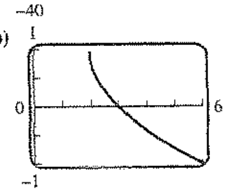
(d)



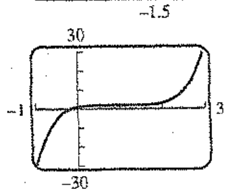
19. (a)



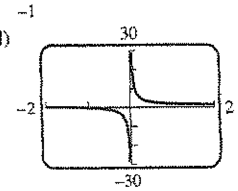
(b)



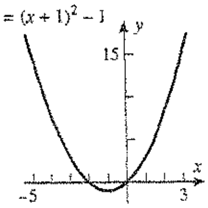
(c)



(d)



21. $y = (x+1)^2 - 1$

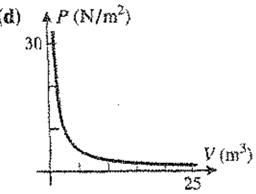


23. (a) newton-meters (N·m) (b) 20 N·m

(c)

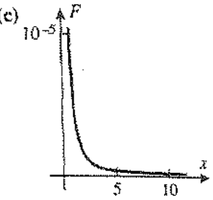
V (L)	0.25	0.5	1.0	1.5	2.0
P (N/m²)	80×10^3	40×10^3	20×10^3	13.3×10^3	10×10^3

(d)

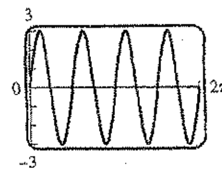


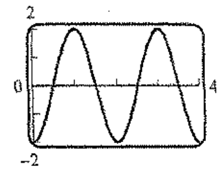
25. (a) $k = 0.000045$ N·m² (b) 0.000005 N (d) The force becomes infinite; the force tends to zero.

(c)

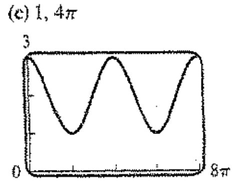


27. (a) II; $y = 1, x = -1, 2$ (b) I; $y = 0, x = -2, 3$ (c) IV; $y = 2$ (d) III; $y = 0, x = -2$
29. (a) $y = 3\sin(x/2)$ (b) $y = 4\cos 2x$ (c) $y = -5\sin 4x$
31. (a) $y = \sin[x + (\pi/2)]$ (b) $y = 3 + 3\sin(2x/9)$ (c) $y = 1 + 2\sin\left[2\left(x - \frac{\pi}{4}\right)\right]$
33. (a) $3, \pi/2$ (b) $2, 2$





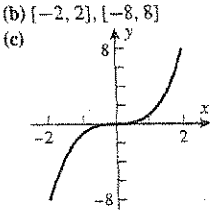
(c) $1, 4\pi$



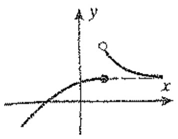
35. (b) $A = \sqrt{A_1^2 + A_2^2}, \theta = \tan^{-1}(A_2/A_1)$ (c) $x = \frac{5\sqrt{13}}{2}\sin(2\pi t + \tan^{-1}\frac{1}{2\sqrt{3}})$

▶ Exercise Set 1.5 (Page 62)

1. (a) yes (b) no (c) yes (d) no
3. (a) yes (b) yes (c) no (d) yes (e) no (f) no
5. (a) yes (b) no
7. (a) $8, -1, 0$ (b) $[-2, 2], [-8, 8]$ (c)



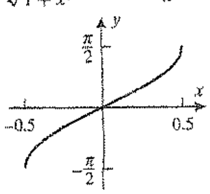
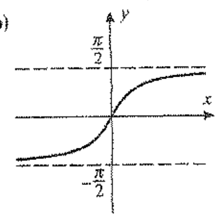
9. $\frac{1}{7}(x+6)$
11. $\sqrt[3]{(x+5)/3}$
13. $(x^3+1)/2$
15. $-\sqrt{3/x}$
17. $\begin{cases} (5/2) - x, & x > 1/2 \\ 1/x, & 0 < x \le 1/2 \end{cases}$
19. $x^{1/4} - 2$ for $x \ge 16$
21. $\frac{1}{2}(3 - x^2)$ for $x \le 0$ 23. $\frac{1}{10}(1 + \sqrt{1 - 20x})$ for $x \le -4$
25. (a) $y = (6.214 \times 10^{-4})x$ (b) $x = \frac{10^4}{6.214}y$ (c) how many meters in y miles
27. (b) symmetric about the line $y = x$ 29. 10

33. 

35. $\frac{4}{5}, \frac{3}{5}, \frac{3}{4}, \frac{5}{3}, \frac{5}{4}$

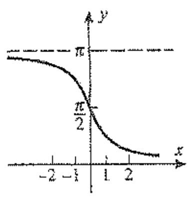
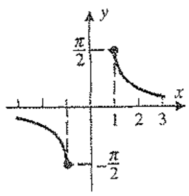
37. (a) $0 \le x \le \pi$ (b) $-1 \le x \le 1$ (c) $-\pi/2 < x < \pi/2$ (d) $-\infty < x < +\infty$ 39. $\frac{24}{25}$

41. (a) $\frac{1}{\sqrt{1+x^2}}$ (b) $\frac{\sqrt{1-x^2}}{x}$ (c) $\frac{\sqrt{x^2-1}}{x}$ (d) $\frac{1}{\sqrt{x^2-1}}$

43. (a)  (b) 

45. (a) $x = 3.6964$ rad (b) $\theta = -76.7°$

47. (a) 0.25545, error (b) $|x| \le \sin 1$

49. (a) $\cot^{-1}(x)$  $\csc^{-1}(x)$ 

(b) $\cot^{-1} x$: all x, $0 < y < \pi$
$\csc^{-1} x$: $|x| \ge 1$, $0 < |y| < \pi/2$

51. (a) 55.0° (b) 33.6° (c) 25.8° 53. (a) 21.1 hours (b) 2.9 hours

55. 29°

Exercise Set 1.6 (Page 74)

1. (a) -4 (b) 4 (c) $\frac{1}{4}$ 3. (a) 2.9690 (b) 0.0341

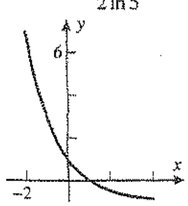
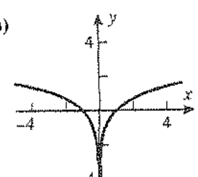
5. (a) 4 (b) -5 (c) 1 (d) $\frac{1}{2}$ 7. (a) 1.3655 (b) -0.3011

9. (a) $2r + \frac{s}{2} + \frac{t}{2}$ (b) $s - 3r - t$

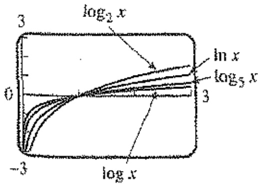
11. (a) $1 + \log x + \frac{1}{2}\log(x-3)$ (b) $2\ln|x| + 3\ln\sin x - \frac{1}{2}\ln(x^2+1)$

13. $\log\frac{256}{3}$ 15. $\ln\frac{\sqrt[3]{x}(x+1)^2}{\cos x}$ 17. 0.01 19. e^2 21. 4 23. 10^5

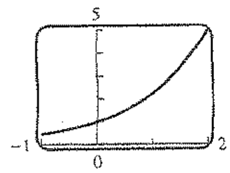
25. $\sqrt{3/2}$ 27. $-\frac{\ln 3}{2\ln 5}$ 29. $\frac{1}{3}\ln\frac{7}{2}$ 31. -2 33. $0, -\ln 2$

35. (a)  (b) 

37. 2.8777, -0.3174

39. 

41. $x \approx 1.471, 7.857$

43. (a) no (b) $y = 2^{x/4}$ (c) $y = 2^{-x}$ (d) $y = (\sqrt{5})^x$ 

45. $\log\frac{1}{2} < 0$, so $3\log\frac{1}{2} < 2\log\frac{1}{2}$ 47. 201 days

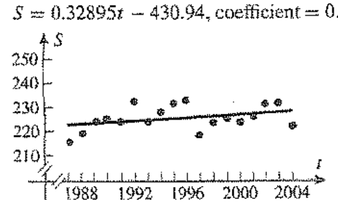
49. (a) 7.4, basic (b) 4.2, acidic (c) 6.4, acidic (d) 5.9, acidic

51. (a) 140 dB, damage (b) 120 dB, damage (c) 80 dB, no damage (d) 75 dB, no damage

53. ≈ 200 55. (a) $\approx 5 \times 10^{16}$ J (b) ≈ 0.67

Exercise Set 1.7 (Page 82)

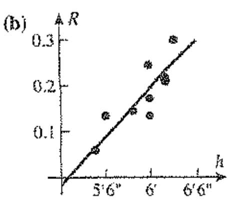
1. II 3. $S = 0.32895t - 430.94$, coefficient $= 0.3433$



5. (a) $p = 0.0146T + 3.98$, 0.9999 (b) 3.25 atm (c) $\approx -272°C$

7. (a) $R = 0.00723T + 1.55$ (b) $\approx -214°C$

9. (a) $S = 0.50179w - 0.00643$ (b) ≈ 16.0 lb

11. (a) $R = 0.2087h - 1.0549$, 0.842333 (b) 

13. (a) $3b^2 + 2b + 1$ (b) $y = x - 1/3$

15. The linear regression line joins the third point to the midpoint of the vertical line segment between the other two points.

17. (a) 181.8 km/s/Mly (b) 1.492×10^{10} years (c) increase

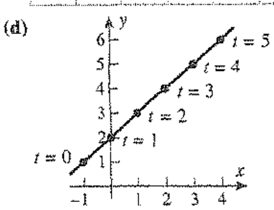
19. $T = 849.5 + 143.5\sin\left[\frac{\pi}{183}t - \frac{\pi}{2}\right]$ 21. $t = 0.445\sqrt{d}$

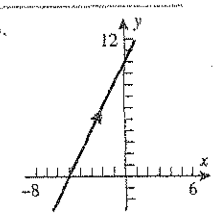
Exercise Set 1.8 (Page 93)

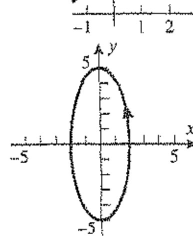
1. (a) $y = x + 2$

(c)

t	0	1	2	3	4	5
x	-1	0	1	2	3	4
y	1	2	3	4	5	6

(d) 

3. 

5. 

7.